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On the presentation of wave phenomena of electrons with the Young–Feynman experiment

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Abstract

The Young–Feynman two-hole interferometer is widely used to present electron wave–particle duality and, in particular, the buildup of interference fringes with single electrons. The teaching approach consists of two steps: (i) electrons come through only one hole but diffraction effects are disregarded and (ii) electrons come through both holes and interference fringes are described. Therefore, a student might believe that wave phenomena are not revealed in case (i), but they arise only by the combined effect of electrons from the two holes. To avoid misunderstanding regarding the distribution of electrons passing through one hole, Fresnel and Fraunhofer diffraction patterns are discussed. In particular, an original experiment, realized with a standard electron microscope and a sample with round holes, is presented to introduce the wave nature of electrons. The experimental results clearly show that a careful discussion of electron diffraction phenomena from one hole provides students with the evidence that the interference experiment from both holes is not strictly required to show the superposition of electron waves.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The double slit interference experiment is usually adopted to discuss the superposition principle applied to radiation and to show the peculiar behaviour of material particles. The dual-slit experiment was used by Einstein and Bohr to develop their considerations on the basic concepts of quantum mechanics. Subsequently, Feynman also made an extensive use of the Young interferometer to describe the mysterious behaviour of particles [1]. More recently, a number of experiments have been realized to show not only the wave nature of electrons (see [2] and references therein), but also of neutrons [3] and more massive particles [4]. Today, a number of books, short movies and lectures on the web try to popularize the mysterious

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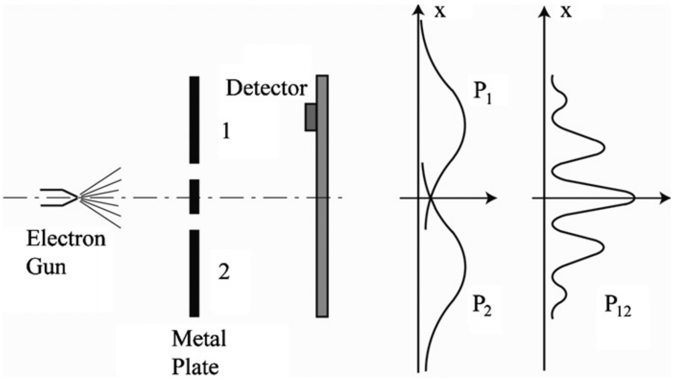


Figure 1. Schematic arrangement of the double-hole experiment to observe electron interference.

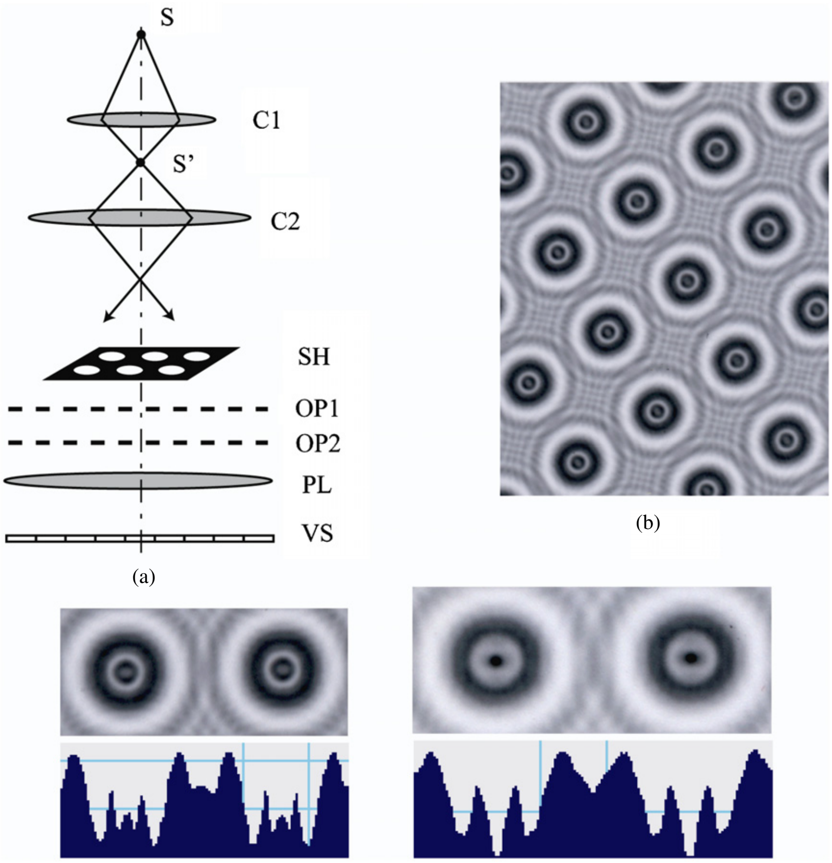
behaviour of electrons on the basis of the Feynman approach, which consists of a Young two-hole interferometer equipped with a detector to reveal single electrons. Firstly, all these presentations consider the case in which one hole is blocked so that electrons go through the other hole. The electron distribution, revealed by the detector, is represented by a Gaussian-like curve as in the case where the same experiment is realized with bullets or water waves. Subsequently, electrons are allowed to pass through both holes of the interferometer and, as a result, an interference fringe system is formed. As a consequence, a student may infer that wave phenomena arise only by the combined effect of the two holes. However, it must be recalled that electrons are diffracted from a single aperture as theoretically discussed by Feynman himself in [1] and by many other authors (see for example [5]). It turns out that the wave nature of electrons can be presented to students either with the formation of diffraction effects at a single hole or with interference patterns from a two-hole setup.

The aim of this paper is to recall attention to wave phenomena which take place in the study of the mysterious behaviour of electrons and, in particular, to those electrons going through one of the interferometer holes. As with optics, the Fresnel and Fraunhofer diffraction conditions are discussed. An original experiment with electrons is presented to emphasize the subtleties that students encounter concerning the formation of the Fresnel diffraction and interference patterns. Subsequently Fraunhofer diffraction of electrons is expounded taking into account the undergraduates basic knowledge of optics. From these considerations it turns out that the description of the buildup of an interference pattern from two holes is not strictly required to reveal the quantum behaviour of electrons.

2. Observation of wave phenomena of electrons using the Young–Feynman interferometer

Figure 1 shows a version of the Young interferometer adopted by Feynman to introduce the interference of electron waves. The interferometer consists of a source that fires electrons towards a thick metal plate with two holes 1 and 2. At a long distance with respect to the separation between the two holes, a backstop is placed with a movable detector which is connected to a loudspeaker.

According to Feynman [1]: . . . ‘As we move the detector around, the rate at which the clicks appear is faster or slower, but the size (loudness) of each click is always the same’, so that ‘. . . whatever arrives at the backstop arrives in “lumps”’ . . . ‘Electrons always arrive in



(a) (b) (c) (d)

Figure 2. (a) Basic ray diagram for the formation of near-field diffraction patterns with electrons; (b) diffraction pattern, at the plane OP1, of some holes of the sample SH; (c) enlarged view of two of the holes shown in (b) and diagram of the related intensity and (d) enlarged view of the diffraction pattern, and plot of the intensity, of two holes of SH formed at the plane OP2.

identical lumps'. Subsequently Feynman asks the question: 'What is the relative probability that an electron "lump" will arrive at the backstop at various distances x from the centre?' and Feynman's answer is: 'The probability that lumps will arrive at a particular x is proportional to the average rate of clicks at that x'. Moreover, P_1 is the probability distributions for electrons coming through hole 1 while hole 2 is blocked off and vice versa for P_2. It must be underlined that, in a section of his book, Feynman reports that a beam of particles, when it goes through a slit, suffers a spreading out, or diffraction, just as for light. Therefore, students must be stimulated to consider that diffraction effects are revealed for electrons passing through one hole of the interferometer in figure 1 and, as a consequence, curves P_1 and P_2 must be represented accordingly.

Two experimental conditions, the so-called near-field (Fresnel) regime and the far-field (Fraunhofer) regime, are reported. Let us consider first the near-field regime. This condition can be obtained, for example, by placing the detector in figure 1 in a plane near the two holes and keeping the distance between the gun and the holes constant. This experiment is realized with the setup in figure 2(a). The basic apparatus is a transmission electron microscope

Philips EM400T, equipped with a hairpin-shaped tungsten wire which forms a primary electron source S. The multiple-position rotary switch of the condenser lens C1 provides a range of electron spot sizes S' which are smaller than the primary source S. S' acts as the effective electron source. The excitation of the second condenser system C2 is changed in such a way that an electron beam, with a divergence down to $(10^{-5}–10^{-6})$ radians, is directed to a sample SH. It consists of a commercially available carbon film (15–20 nm thick) which has regular arrays of round holes (hole spacing 2.1 μm, hole diameter 1.2 μm). The objective lens of the microscope (not represented in figure 2(a)) is switched off and the diffraction mode is selected. The projector lens system PL provides real and magnified images of the electron distribution at the observation plane OP1 or OP2 onto the viewing screen VS or onto a detector placed just below VS. The Fresnel diffraction patterns in the plane OP1 or OP2, that is at two different distances from SH, are obtained by changing the focus control of the diffraction lens included in the PL system. The micrographs were recorded on photographic plates with an exposure time of 150 s.

It must be emphasized that, in analogy with optics, the agreement between scalar diffraction theory and experiments has been demonstrated for electron waves [6–9]. In particular, the intensity and phase distributions have been derived for the Fresnel diffraction patterns both from a circular aperture in an opaque screen [6–9] and from a thin wire [10].

Figure 2(b) reports the near-field diffraction pattern of some holes formed in the plane OP1. Figure 2(c) shows an enlarged view of two holes in figure 2(b) together with the intensity profile. Figure 2(d) reports a magnified image of the diffraction pattern, and the intensity plot of two holes formed on the plane OP2, at a longer distance from SH. The following features of the patterns in figures 2(c) and (d) are worth noting: (i) in figure 2(c), a maximum of intensity is detected in the centre of the diffraction patterns because an odd number of Fresnel zones are transmitted by the holes. In figure 2(d), the phase contribution from an even number of Fresnel zones through the apertures produces a minimum of intensity in the centre of the diffraction patterns. According to previous experimental results [6, 7, 9], when only one hole of our sample can be selected, the resulting diffraction pattern will not differ significantly from those of each hole in figures 2(c) or (d). Therefore, the intensity profiles of the diffraction effects at one hole must replace curves P_1 and P_2 in figure 1, moreover, these plots form at symmetrical positions with respect to $x = 0$; (ii) a pattern, arising from the overlapping of the intensity distribution of the waves diffracted by both holes, is revealed in the area between the two holes and is centred at a position which corresponds to $x = 0$ in figure 1.

A situation more familiar to students is the Fraunhofer diffraction pattern obtained by placing the detector at a greater distance from the two holes (for a detailed discussion see [7] and references therein). Figure 3 reports an experimental diffraction pattern of electrons passing through a circular hole [11], for example hole 1 in figure 1. The same happens when hole 2 is covered. Therefore, both P_1 and P_2 in figure 1 have to represent the intensity of diffraction effects.

Moreover, students should bear in mind that the maxima of P_1 and P_2 superpose on the optical axis in figure 1 at $x = 0$ and not at a value of x which is on a straight line with the electron gun and hole 1, and hole 2, respectively.

3. Concluding remarks

The considerations developed above emphasize that the study of the wave nature of electrons should be introduced once students have a very good knowledge of light wave phenomena, in particular of diffraction effects from a slit or a circular aperture. It must be recalled that *diffraction effects* from one hole, either in the Fraunhofer or Fresnel regime, are calculated by

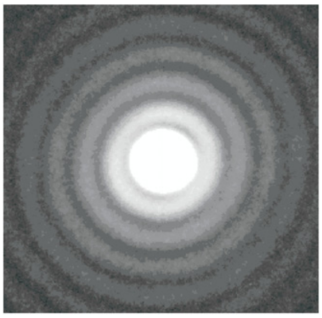


Figure 3. Fraunhofer diffraction pattern at a circular hole (diameter 10 μm).

applying the superposition principle (*interference*) to a large number of elementary sources. Therefore the sentence *diffraction from one hole* could be replaced by *interference from one hole*.

In this paper, an original electron experiment has been realized to show Fresnel's diffraction phenomena. Although a low coherent source is used, diffraction and interference effects from a few holes are revealed so that the wave phenomena of electrons are successfully presented to students without resorting to dedicated interferometers. Experimental evidence of Fraunhofer diffraction has also been reported. Students should remember that curves P_1 and P_2 in figure 1 take place at $x = 0$ and not off optical-axis as in the Fresnel regime. Moreover, P_1 and P_2 represent the intensity of a diffraction pattern of a round hole, as reported in figure 3, and must be drawn accordingly. Since diffraction effects show clearly the wave-like behaviour of electrons, a discussion about electrons going through both holes to reveal wave phenomena is not strictly necessary. Alternatively, the interference of electron waves can be shown directly to students only with those electrons coming through two holes. However, a detailed discussion of the detected effects must also consider the modulation of the interference pattern by diffraction from one hole.

As demonstrated by preliminary measurements realized in our laboratory [12], the electron source and optical conditions used to record the images in figure 2 have been arranged to obtain a beam which is made up of single electrons travelling at various intervals along the interferometer. The buildup of the wave patterns in figures 2(c) and (d) are, thus, a convincing demonstration of the wave behaviour of *single* electrons. Work is in progress along these lines.

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References

[1] Feynman R P, Leighton R B and Sands M 1965 *The Feynman Lectures on Physics* vol 3 (Menlo Park, CA: Addison-Wesley)
[2] Matteucci G, Castaneda R, Serna S, Medina F and Garcia-Sucerquia J 2010 Discovering the puzzling behaviour of electrons with the Grimaldi–Young experiment *Eur. J. Phys.* **31** 347–56
[3] Gaehler R and Zeilinger A 1991 Wave-optical experiments with very cold neutrons *Am. J. Phys.* **59** 316–24

[4] Nalrz O, Arndt M and Zeilinger A 2003 Quantum interference experiments with large molecules *Am. J. Phys.* **71** 319–25

[5] Eisberg R and Resnick R 1985 *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles* (New York: Wiley)

[6] Komrska J 1971 Scalar diffraction theory in electron optics *Advances in Electronics and Electron Physics* vol 30 (New York: Academic) pp 139–234

[7] Matteucci G 2005 Electron holography used to realize Young's double-hole experiment together with the mapping of magnetic fields *Eur. J. Phys.* **26** 481–9

[8] Medina F F, Sucerquia J G, Castaneda R and Matteucci G 2004 Angular criterion to distinguish between Fresnel and Fraunhofer diffraction *Optik* **115** 547–52

[9] Matteucci G 1990 Electron wavelike behavior: a historical and experimental introduction *Am. J. Phys.* **58** 1143–7

[10] Matteucci G, Castaneda R, Serna S, Medina F and Garcia-Sucerquia J 2010 Discovering the puzzling behaviour of electrons with the Grimaldi–Young experiment *Eur. J. Phys.* **31** 347–56

[11] Matteucci G, Ferrari L and Migliori A 2010 The Heisenberg uncertainty principle demonstrated with an electron diffraction experiment *Eur. J. Phys.* **31** 1287–93

[12] Balbi G, Berti R, Costa A, Gabrielli A, Giorni F, Matteucci G, Pozzi G, Semproni N, Villa M and Zoccoli A 2010 Esperienza di interferenza di singolo elettrone con doppia fenditura e sensore ad alta risoluzione temporale Abstract on *Proc. Società Italiana di Fisica—XCVI Congresso Nazionale (20–24 Settembre)* pp 31–2