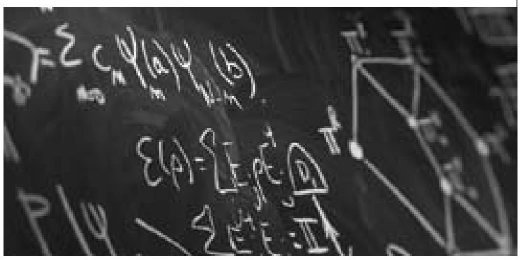
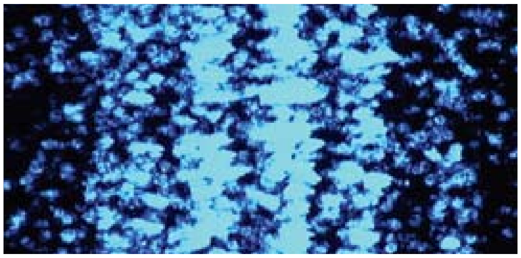
Contents





THE CHALLENGE OF QUANTUM REALITY

PERIMETER EXPLORATIONS

This series of in-class educational resources is designed to help teachers explain a range of important topics in physics. Perimeter Explorations is the product of extensive collaboration between international researchers, Perimeter Institute's outreach staff, and experienced classroom teachers. Each module has been designed with both the expert and novice teacher in mind, and has been thoroughly tested in classrooms.

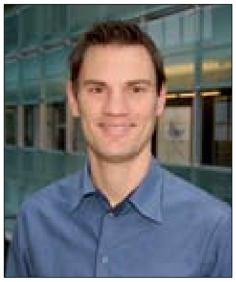
PERIMETER INSTITUTE

Canada's Perimeter Institute for Theoretical Physics is an independent, non-profit, scientific research and educational outreach organization where international scientists gather to push the limits of our understanding of physical laws and explore new ideas about the very essence of space, time, matter, and information. The award-winning research centre provides a multi-disciplinary environment to foster research into Cosmology, Particle Physics, Quantum Foundations, Quantum Gravity, Quantum Information, Superstring Theory, and related areas.

The Institute, located in Waterloo, Ontario, also provides a wide array of educational outreach activities for students, teachers, and members of the general public in order to share the joy of scientific research, discovery, and innovation. Additional information can be found online at **www.perimeterinstitute.ca.**

YOUR NARRATOR

Dr. Damian Pope is Senior Manager of Scientific Outreach at Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada. He holds a PhD in theoretical physics from the University of Queensland, Australia, and his area of specialty is quantum physics. He also has extensive experience in explaining the wonders of physics to people of all ages and from all walks of life.



Introduction

Quantum physics has revolutionized our understanding of nature and helped catapult humanity into the information age. It describes the subatomic world, including protons, neutrons, and electrons, with phenomenal accuracy. Quantum physics gives us deep insight into the building blocks of the universe.

Peering into the quantum world means we have to let go of some of the comfortable, often intuitive, notions we develop from day-to-day living. The laws of classical physics provide a neat and tidy description of the world, which allow us to send astronauts to the Moon but fail to explain how the electronics in a simple calculator work. To design electronics at a fundamental level we must use the laws of quantum physics, which grew out of the analysis of several important experiments.

One of the most important experiments in quantum physics is the double-slit experiment. In this experiment, individual quantum objects, such as electrons or photons, are fired at a barrier with two narrow slits. After passing through the slits, they produce an interference pattern on a detector screen on the other side of the barrier. This result leads to one of the deepest mysteries of quantum physics—*wave–particle duality*—the fact that electrons and other quantum objects behave like waves in some situations and like particles in others.

Wave–particle duality stands in contrast to the everyday world of classical physics. In the classical picture, things are described using either a wave model or a particle model. But, in the quantum world things can be described as both wave and particle. The neat and tidy models of classical physics are blurred in the quantum world.

This resource is aimed at senior high school students. It introduces quantum physics and wave–particle duality using the double-slit experiment with electrons. Students will discover some of the essential features of quantum physics, including the de Broglie wavelength

$$\lambda = \mathrm{h}/p$$

and the equation for the energy of a photon

$$E = \mathrm{h}f$$

The video provides an opportunity to experience the strange quantum world firsthand. First, it supplies necessary background information by exploring the contrasts between classical particles and classical waves in the double-slit experiment. It then looks at nature on a quantum scale by investigating the behaviour of electrons in the double-slit experiment. This experiment yields results that defy classical thinking. Electrons leave the source as particles and strike the detection screen as particles, producing small localized dots. However, a distinctive interference pattern associated with waves emerges after enough electrons have passed through the apparatus.

In the double-slit experiment with electrons, the intensity of the electron beam can be turned down so that there is only one electron passing through the apparatus at a time—but an interference pattern still develops! If that is not surprising enough, when we try to measure which slit each electron passes through, the interference pattern disappears. This same bizarre behaviour is demonstrated by all other quantum objects, such as neutrons, photons and even large molecules.

Students will feel the sense of excitement that rippled through the 1920s scientific community during the early years of quantum theory. After grappling with wave–particle duality, students will be led through some of the intriguing questions under debate. They will experience the process of science as they are presented with the reality that there is no consensus among physicists as to what electrons are actually doing in the double-slit experiment. This lack of consensus has led to several competing interpretations. Students will be introduced to four of these interpretations by leading experts in the field.

The video ends with a snapshot of the power of science. Despite the lack of consensus about the exact nature of the electron, our ability to manipulate it using the laws of quantum physics has led to a technological revolution that has shaped the world we live in. Quantum physics is used in everyday technologies, from lasers, LEDs, and solar cells to the heart of the information age—the computer. Understanding and applying quantum physics has dramatically increased our ability to generate and share knowledge. The next generation of innovators are exploring quantum computers that hold the promise of wildly exceeding our current abilities.

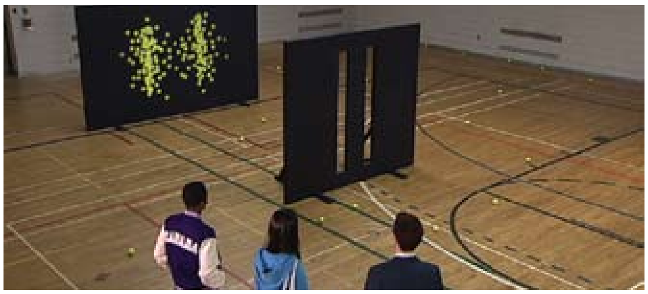
Curriculum Links

Topic	Connection to Quantum Physics	Relevant Materials
Classical Particle Behaviour	Classical particles are localized and follow predictable trajectories. When two particles collide their trajectories are changed.	Video: Chapter 1 Worksheets 1, 2, and 5
Classical Wave Behaviour	Classical waves are non-localized and transfer energy as they propagate. When two waves overlap they interfere, generating maxima and minima.	Video: Chapter 1 Worksheets 1, 2, and 5
Wave–Particle Duality of Electrons	Electrons exhibit both wave and particle behaviour. In the double-slit experiment electrons are detected as localized particles that produce an interference pattern.	Video: Chapter 2 Worksheets 1, 2, 3, 4, and 5
Wave Nature of Matter	The de Broglie wavelength for matter is given by $\lambda = \mathrm{h}/p$	Video: Chapters 2 and 3 Worksheets 1, 2, 3, and 4
Wave–Particle Duality of Light	Single photon experiments demonstrate that light also exhibits both wave and particle behaviour.	Video: Chapter 3 Worksheets 1, 2, and 3
Quantization of Light Energy	A light particle is called a photon and has energy $E = \mathrm{h}f$	Video: Chapter 3 Worksheets 1, 2, 3, and 4
Measurement Disturbance	In the electron double-slit experiment, measuring which slit the electrons pass through destroys the interference pattern.	Video: Chapter 4 Worksheets 1 and 2
Scientific Models	There are several competing models, or interpretations, that attempt to explain what an electron actually is, and what it is doing during the double-slit experiment.	Video: Chapter 5 Black Box Demonstration Worksheets 1, 2, and 5
Applications of Wave–Particle Duality	Using the laws of quantum physics has led to technological innovations that have changed society. The most dramatic example is the transistor, which is at the heart of the computer and the Information Age.	Video: Chapter 5 Worksheets 1, 2, and 3

Quantum Physics
In a Nutshell

CLASSICAL PHYSICS

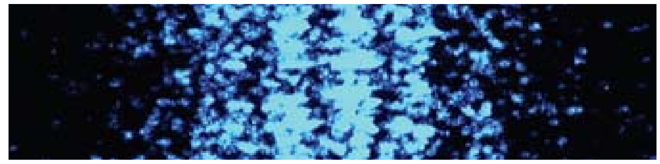
- Classical physics is the physics of the motion, energies, and interactions of objects in the everyday world around us.
- In the double-slit experiment, tennis balls and all other *classical particles* move as localized particles through the slits and once they hit the screen they produce the following distribution:



- If we use water waves, sound, or any other *classical waves* they spread out behind the double-slit barrier and produce an *interference pattern*.
- Light also spreads out behind the double-slit barrier and produces an interference pattern.

QUANTUM PHYSICS

- Quantum physics is revealed in the physics of isolated processes, typically with very small subatomic objects.
- In the electron double-slit experiment, each electron hits the detection screen as a particle.
- After many electrons hit, an interference pattern forms, demonstrating wave behaviour.
- The same interference pattern forms even when we fire electrons one at a time.



- These results show that electrons exhibit both wave and particle behaviour, i.e., *wave–particle duality*.
- The de Broglie wavelength describes the wave behaviour of particles such as electrons. It is given by the equation

$$\lambda = \mathrm{h}/p$$

- Light also exhibits wave–particle duality. In the double-slit experiment light hits the detection screen as an individual particle, but over time it forms an interference pattern like a wave.
- A particle of light is called a *photon* and its energy is given by

$$E = \mathrm{h}f$$

- All quantum objects, including protons, neutrons, atoms, and molecules, exhibit wave–particle duality.
- When we look at the electron to see what it is doing while passing through the double-slit barrier, we are making a measurement which perturbs the electron and destroys the interference pattern. This demonstrates *measurement disturbance*.
- We can predict the overall behaviour of the electrons in the double-slit experiment, but nobody really knows what the electrons are doing between the source and the detector. To complete the picture, physicists have proposed various interpretations, including:

  i) thinking of electrons as spread-out waves that collapse to point-like particles once they are measured (Collapse Interpretation),
  ii) thinking of electrons as particles that are guided by an invisible wave (Pilot Wave Interpretation),
  iii) thinking of parallel universes that come into being when we make measurements at the quantum level (Many Worlds Interpretation)
  iv) thinking exclusively about the direct results of measurements (Copenhagen Interpretation).

- In spite of these differing views, quantum physics plays a crucial role in a number of everyday technologies including computers, remote control devices, lasers, and cell phones.

Suggested Ways To Use This Resource

This flexible resource includes a classroom video, five student worksheets, and a hands-on demonstration. The worksheets are provided in editable electronic form so that you can modify them as you wish.

OUTLINE FOR A SINGLE PERIOD

Black Box Demonstration (10 minutes)
Video (25-35 minutes). You may wish to pause the video between chapters to discuss questions.
Discussion (15 minutes): Worksheet 2 *Concept Questions*
Homework: Worksheet 1 *Summary Questions* or Worksheet 3 *Mathematical Investigation*

OUTLINE FOR TWO PERIODS

FIRST CLASS

Black Box Demonstration (10 minutes)
Activity (45 minutes): Worksheet 5 *Investigating the Nature of the Electron*
Homework: Worksheet 5 *Investigating the Nature of the Electron* Summary Questions

SECOND CLASS

Review activity results (10 minutes)
Video (25-35 minutes): You may wish to pause the video between chapters to discuss questions.
Discussion (15 minutes): Worksheet 2 *Concept Questions*
Homework: Worksheet 3 *Mathematical Investigation*

ENRICHMENT OPTIONS

Worksheet 4 *Advanced Mathematical Analysis*
Discussion: Provide students with the Chapter 5 summary *Interpretations and Applications.* Discuss the choices and consequences of each interpretation.

Student Activities

DEMONSTRATION

The goal of this exercise is to show students that several different models can be created from one set of observable data and that each model is equally acceptable if it predicts the observed results.

Students design a simple pencil and paper model to describe what *might* be happening inside the black-box device. They construct their model of what is happening inside the tube from data gathered from outside the tube. Students then present their ideas to their peers.

WORKSHEETS

Worksheet 1: Video summary questions including conceptual short answer and mathematical questions.

Worksheet 2: Concept questions intended to be used to stimulate discussion and deepen student understanding. Students will get the most benefit if they are given an opportunity to choose an answer, share their thinking with a peer, hear what others are thinking, and then have an opportunity to change their minds before hearing the correct answer.

Worksheet 3: Mathematical investigation of wave–particle duality

Worksheet 4: Advanced mathematical questions designed as an enrichment exercise

Worksheet 5: In this activity, students will use the double-slit experiment to investigate the nature of classical objects, classical waves, light, and electrons.

The DVD-ROM contains editable electronic copies of all the worksheets.

Black Box Demonstration

The goal of this exercise is to show students that several different models can be created from one set of observable data and that each model is equally acceptable if it predicts the observed results.

Students design a simple pencil and paper model to describe what might be happening inside the black box device. They construct their model of what is happening inside the tube from data gathered from outside the tube. Students then present their ideas to their peers.

MATERIALS

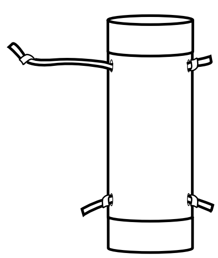
black box device
(see Appendix A for building instructions)

METHOD

01. Arrange the black box so one of the top cords is extended. Pull the other top cord across so that it is now extended. Pull the top two cords back and forth a few times, ensuring that all students can see the apparatus. Each time you pull one cord, the other will retract into the black box. This will give the students the sense that the top two cords are, in fact, only one cord. Then pull one of the bottom two cords. Students will be surprised to see that pulling a bottom cord also causes a top cord to retract. Continue the demonstration by randomly pulling each cord. Ask students to call out predictions as you pull on the cords.

02. Allow students to try their own combinations, noting the motion and tension of the cords or anything else that might help them decipher how the cords are attached.

03. Now instruct students to complete this sketch by drawing their interpretation of how the cords might be attached. Make sure students do this individually.



04. Have several students share their sketches on the board.

05. Systematically test the accuracy of each student's idea, analyzing the diagram to see if it *could* predict the behaviour that is actually witnessed when the cords are pulled. Each drawing will likely have at least a chance of working. This demonstration is most effective when there are at least 10 different drawings on the board, each with the potential of being the correct depiction of how the cords are actually attached. Refine the models drawn on the board by drawing out new predictions (e.g., Would the black box make a noise if it is shaken?).

Note: Never divulge how the black box device is actually connected. The models must be judged primarily on their ability to explain and predict the observations.

DISCUSSION

Scientists use models to represent or simplify complex realities. Sometimes the models are so good at representing the reality that we forget they are models. Sometimes the reality is so complex that a simple physical model is inadequate.

One of the challenges arising from quantum physics is creating simple models that make sense in the classical world while remaining true to the reality of the quantum world. Wave–particle duality is an example of this challenge. There are no classical analogues that can accurately represent the behaviour of a quantum object, so we are left with a model that does not really make sense in the classical world.

Another challenge arising from quantum physics is that a model is measured primarily by its ability to explain the observations, and observing quantum systems can be problematic. There are real limits to what can and cannot be measured in the quantum world and, therefore, a limit to how refined the models can be. In the video we explore the various models, or interpretations, for what is happening during the double-slit experiment. Some interpretations may be preferred over others because they provide useful insights or make different assumptions, but each of them provides a complete description of the observed data. Any attempt to observe what the object is doing during the double-slit experiment alters the data and prevents us from refining our models.

SUGGESTED USES

Pre-video: Use the activity to introduce a discussion about how physicists construct models. Draw out the concept that models are built in response to observations and should have predictive power.

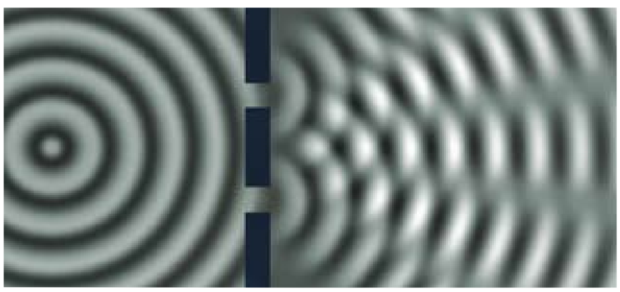
Post-video: Use the activity to revisit the part of the video where scientists provide alternative interpretations for the double-slit experiment. Build on the analogy that the inside of the black box is like the "inside" of the double-slit experiment. We cannot know what is "actually" happening inside, so any model that successfully explains the observed data can be considered valid.

Worksheet 01:
Video Summary

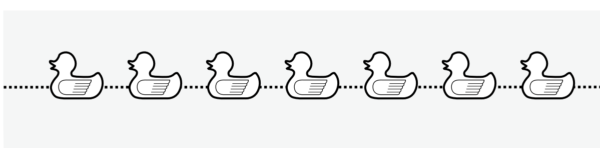
Useful
equations: $\lambda = h/p$ $E = hf$

01. Baseballs are fired at a barrier with two narrow slits. Behind the barrier is a wall. Draw a distribution that shows where the baseballs hit the wall.

02. A water wave passes a two-slit barrier, as shown below, generating an interference pattern.

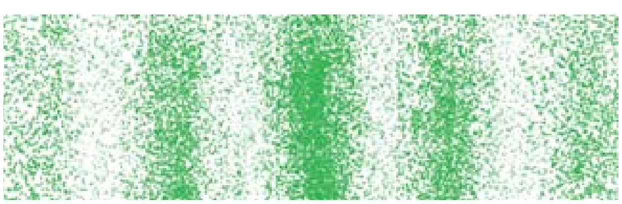


(a) Imagine a rubber duck is floating at each maxima and minima along the reference line shown. Using the diagram below, draw a vertical line for each duck that will show how its vertical position changes over time.



(b) The relative height each rubber duck moves is related to the amount of energy passing at that point. A longer vertical line represents more energy than a shorter line. Describe where the energy is greatest. How does the energy distribution between maxima mimic the energy distribution in a double-slit interference pattern for light?

03. The photograph below shows an interference pattern from the electron double-slit experiment.



(a) The distance between neighbouring interference maxima is 120 μm. Why is this distance so much smaller than the distance between maxima for water waves?
(b) What aspects of the image illustrate the particle nature of electrons?
(c) What aspect of the image illustrate the wave nature of electrons?
(d) How can an electron be a particle and a wave at the same time? Spend a few minutes formulating your explanation for what is going on and then discuss it with your neighbour.

04. The double-slit experiment is performed using light with a wavelength of 580 nm. The light's intensity is so low that only one photon passes through the slits each second. This means no two photons ever interact with each other in the experiment.

(a) What is the energy of each photon emitted?
(b) What aspects of this experiment demonstrate the particle nature of light?
(c) What aspects of this experiment demonstrate the wave nature of light?

05. One of the largest objects that physicists have used to produce an interference pattern is a molecule called PFD (perfluoroalkyl-functionalized diazobenzene, $C_{30}H_{12}F_{30}N_2O_4$). It has a mass of 1.7×10^{-24} kg. In the experiment, the molecule had a de Broglie wavelength of 2.8×10^{-12} m. Calculate the molecule's velocity.

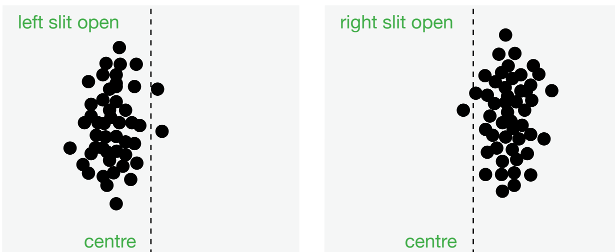
06. What happens to the interference pattern created in the electron double-slit experiment when detectors are used to determine which slit an electron is passing through? How do the researchers explain this result?

07. You are discussing the electron double-slit experiment with a friend. She says: "Physicists understand the experiment completely. Each electron leaves the source as a classical particle and hits the screen as a classical particle. All researchers agree that an electron is a classical particle in the experiment." Write a three to four line reply to your friend that explains why she is mistaken.

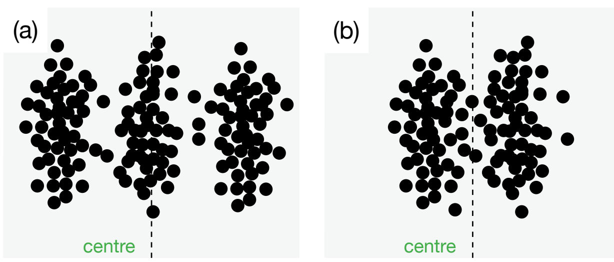
08. Quantum physics is part of your everyday life. List at least five of the technological applications discussed in the video.

Worksheet 02:
Concept Questions

01. Tennis balls are sent toward two slits. The distributions of the marks they make on a wall on the other side of the barrier when one slit is open are shown below.



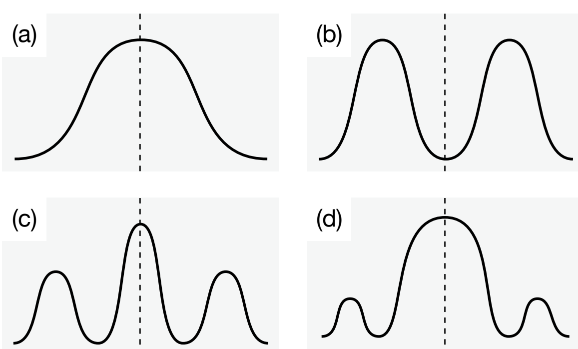
Which distribution would you expect to see if both slits are open at the same time?



02. Which statement correctly describes how waves behave when they occupy the same location at the same time?

(a) A crest overlapping with a crest will constructively interfere to produce a minima.
(b) A crest overlapping with a trough will constructively interfere to produce a minima.
(c) A trough overlapping with a trough will constructively interfere to produce a maxima.
(d) A trough overlapping with a trough will destructively interfere to produce a maxima.

03. A water wave passes through two slits. Which pattern best matches the amplitude of the resulting wave?



04. Classical particles are different from classical waves because classical particles

(a) are spread out and generate an interference pattern in the double-slit experiment.
(b) are localized and generate an interference pattern in the double-slit experiment.
(c) are localized and generate a distribution that is the sum of each single-slit distribution.
(d) are spread out and generate a distribution that is the sum of each single-slit distribution.

05. The video shows the interference of light of a single colour. What would you expect if white light were used?

(a) bands of white light and darkness
(b) bands of different colours of light and darkness
(c) a white central maxima and alternating bands of different colours of light and darkness on either side
(d) no interference pattern

06. To better understand the double-slit experiment, it was important to send electrons through one at a time because

(a) the detector needed time to reset in order to detect the next electron.
(b) the slits were too narrow to allow two electrons to pass at the same time.
(c) this prevented the electrons from interacting with each other.
(d) time is needed to generate more electrons.

07. In the double-slit experiment, electrons

(a) behave like waves and behave like particles.
(b) split in half and go through both slits simultaneously.
(c) behave like particles, but are waves.
(d) are both waves and particles at the same time.

Actual image from the electron double-slit experiment

Worksheet 02:
Continued

08. You get sunburn from ultraviolet light but not from visible light. This is because UV photons have a greater
    (a) mass.
    (b) frequency.
    (c) speed.
    (d) wavelength.

09. Why have interference effects with tennis balls not been observed?
    (a) The de Broglie wavelength equation, $\lambda = h/p$, is only for sub-microscopic objects.
    (b) The experiment has not been done yet.
    (c) The de Broglie wavelength for a tennis ball will be much smaller than for an atom.
    (d) The de Broglie wavelength for a tennis ball will be larger than for an atom.

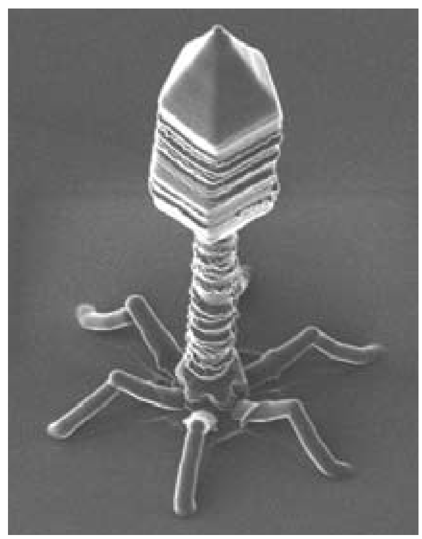
10. All quantum objects exhibit wave–particle duality. In the double-slit experiment this is shown by the fact that individual objects hit the screen
    (a) at specific locations and build up an interference pattern after a large number have hit.
    (b) in a spread-out way and build up an interference pattern after a large number have hit.
    (c) at specific locations and build up a particle distribution after a large number have hit.
    (d) in a spread-out way and build up a particle distribution after a large number have hit.

11. If we do measurements to determine which slit an electron went through, we find that
    (a) half of the electron goes through each slit.
    (b) the whole electron goes through both slits.
    (c) the whole electron goes through one or the other slit.
    (d) it is impossible to detect an electron.

12. With electrons in the double-slit experiment, physicists know
    (a) where an electron will hit the screen.
    (b) which slit the electron went through, without the aid of a detector.
    (c) that the electron went through both slits.
    (d) that all of the interpretations give the same predictions for the overall results.

13. There are competing ideas about what is actually happening between the source and the detector in the double-slit experiment. In which of the interpretations does a single electron go through one and only one slit?
    (a) Pilot Wave and Collapse
    (b) Pilot Wave and Many Worlds
    (c) Collapse and Many Worlds
    (d) Pilot Wave, Collapse, and Many Worlds

14. An electron microscope can produce clearer images of significantly smaller objects than a light microscope can because the electrons have a
    (a) larger frequency.
    (b) smaller size.
    (c) slower speed.
    (d) shorter wavelength.



Electron microscope image of a virus

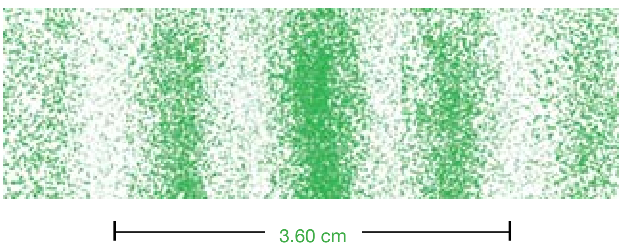
15. Which quantum application has had the greatest effect on your life?
    (a) solar panels
    (b) transistors
    (c) lasers
    (d) other

Worksheet 03:
Mathematical Investigation of Wave-Particle Duality

Useful equations: $\Delta x = \frac{\lambda L}{d}$ $\lambda = \frac{h}{p} = \frac{h}{mv}$ $E = hf$ $V = \frac{E_Q}{q}$ $E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$m_e = 9.11\times10^{-31}$ kg $c = 3.00\times10^{8}$ m/s $h = 6.626\times10^{-34}$ J·s $q_e = 1.602\times10^{-19}$ C   1 amu $= 1.6605\times10^{-27}$ kg

01. The photo below shows the interference pattern produced by an electron double-slit experiment. In this experiment, the electrons were sent through a double-slit apparatus with an effective slit separation of 200 nm. The detector screen was 79.0 cm from the double slits. The image has been magnified by a factor of 100.



(a) Use Young's double-slit equation to determine the wavelength of the electrons.
(b) Use the de Broglie wavelength equation to determine the momentum and velocity for the electrons passing through the apparatus.
(c) The electrons were accelerated by an electric field. Calculate the potential difference needed to produce these results.

02. The resolving power of imaging devices is limited by the wavelength of radiation used. Optical microscopes use visible light, so they can only resolve objects down to a size of about 200 nm. Electron microscopes can resolve much smaller objects because the wavelength of the electrons can be made much shorter than the wavelength of visible light.

(a) A typical transmission electron microscope (TEM) accelerates the electrons through a potential difference of 30 kV. Calculate the velocity of the electrons incident on the sample.
(b) Determine the de Broglie wavelength for these electrons.
(c) Compare the electron wavelength to the wavelength for green light (550 nm).
(d) If resolving power depended only on wavelength, what would the resolving power of this TEM be?
(e) Using the Internet, research the resolving power for a typical electron microscope.

03. A standard He-Ne laser produces about 1.0 mW of light at a wavelength of 633 nm. To create a single-photon interference experiment the laser is shone through a series of filters that reduce the beam to a small fraction of the original number of photons.

(a) Calculate the number of photons produced by the laser every second.
(b) Determine the time taken for the photons to travel 0.30 m from the filters to the detector.
(c) Each filter absorbs 96% of the photons. How many photons per second pass through after seven filters?
(d) Compare the time taken by each photon to travel 0.30 m with the time between successive photons emerging from the final filter (assume the photons are equally spaced). Express your answer as a fraction. This fraction describes the chance that there is more than one photon in flight between the filters and the detector at any one time.

04. The experiment demonstrating interference of buckminsterfullerene, C_{60}, had the molecules moving at 210 m/s. Each molecule has an atomic mass of 720 atomic units and a diameter of 1 nm. The molecules passed through slits with widths of 50 nm and separations of 100 nm. After the slits, the molecules travelled 1.25 m before being detected.

(a) What is the mass of one molecule?
(b) What is the momentum?
(c) What is its wavelength?
(d) How does this wavelength compare with the size of the molecule?
(e) How does this wavelength compare with the size of the slits?
(f) What would the distance between fringe maxima be if the screen was 5.0 m from the slits?

Worksheet 04:
Advanced Mathematical Analysis

Useful equations:
$$E_Q = \frac{kq_1q_2}{r} \qquad E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \qquad \lambda = \frac{h}{p} \qquad E = hf \qquad \varepsilon = \frac{F_Q}{q} \qquad V = \varepsilon d$$

$$\mathrm{k} = 8.99\times10^{9}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}} \qquad \mathrm{h} = 6.626\times10^{-34}\,\mathrm{J\cdot s} \qquad 1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J} \qquad m_{electron} = 9.11\times10^{-31}\,\mathrm{kg}$$

PART 1: INVESTIGATING THE HYDROGEN ATOM

A simple model for the hydrogen atom is shown below. In this model, the electron orbits around the proton in circular shells. The first shell represents the lowest energy level and is called the ground state. When the hydrogen atom is in an excited state, the electron will occupy one of the higher shells. To drop down to a lower energy level the electron must emit a photon that has the same amount of energy as the electron needs to lose.

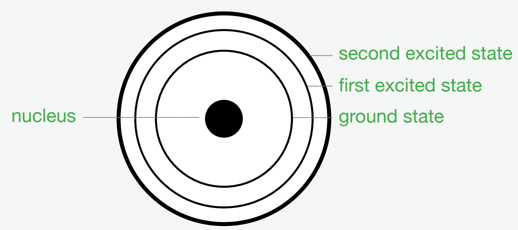


Figure 4.1 Hydrogen atom

01. The ground state for hydrogen is –13.6 eV. The average radius of the ground state is 5.29 x 10^{-11} m. Calculate the wavelength of the electron when it is in the ground state. (Hint: $E_{total} = E_{kinetic} + E_{potential}$)

02. The wavelength of light emitted by the electron when it drops from the second excited state to the ground state is 102.4 nm. Determine the energy of the electron when it is in the second excited state.

03. The average radius of the second excited state is 4.76 x 10^{-10} m. Calculate the wavelength of the electron when it is in this state.

04. The energy levels allowed are those that have integer values for the electron wavelengths. How many complete wavelengths of the electron fit around the orbits of the ground state and the second excited state?

PART 2: FRANCK–HERTZ EXPERIMENT

The Franck–Hertz experiment demonstrates the quantum nature of electrons and light. In this Nobel Prize winning experiment, electrons are accelerated through a low-pressure gas. As the electrons accelerate through the electric field they gain kinetic energy. At very specific distances, we observe bands of monochromatic light. Changing the potential difference, gas pressure, tube length, or gas used will produce changes in the colour or location of the glowing bands.

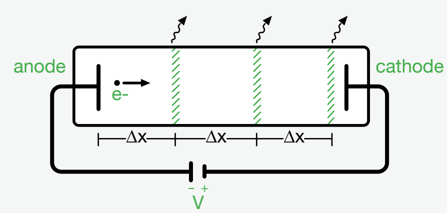


Figure 4.2 The Franck–Hertz experiment

The photons are produced by the excited electrons in the gas molecules when they drop back down after colliding with the accelerated electrons. The energy gained by each electron as it accelerates through the electric field can be determined by

$$E_K = \frac{qV\Delta x}{L}$$

where E_K = the kinetic energy of the electron (J)
q = the charge on the electron (C)
V = the potential difference applied across the tube (V)
Δx = the distance between successive bands (m)
L = the distance from anode to cathode (m)

01. Show how the above equation can be derived from the work done on the electron by the electric field.

02. A Franck–Hertz tube containing low-pressure neon has a potential difference of 22.0 V applied between anode and cathode. The gap is 12.5 mm. The distance from the anode to the first bright band of light (and any successive bands of light) is 1.19 mm.

Worksheet 04:
Continued

(a) Determine the kinetic energy of an electron when it reaches the first bright band.
(b) Calculate the frequency of light being emitted. What wavelength and colour does this equate to?
(c) What happens to the accelerated electron after it loses all of its kinetic energy to the gas molecule? (Hint: Why is there a series of bright monochromatic bands?)
(d) What will happen to the spacing between the light bands if the voltage is doubled?

03. The neon gas is replaced with mercury. When 13.4 V is applied across the 12.5 mm gap between anode and cathode, the wavelength of light emitted is 253 nm. How many bands will be produced? (Note: these bands would not be visible because 253 nm is outside the visible spectrum.)
04. What does the Franck–Hertz experiment tell us about the structure of the atom?
05. What does the Franck–Hertz experiment tell us about the quantum nature of light?

PART 3: INVESTIGATING THE LIMIT OF QUANTUM OBSERVATION

$$\lambda = \frac{h}{mv} \ (1) \qquad \lambda = \frac{\Delta x d}{L} \ (2) \qquad E_k = \frac{1}{2}mv^2 \ (3) \qquad E_k = \frac{1}{2}k_B T_{drift} \ (4)$$

Boltzmann's constant, $k_B = 1.38 \times 10^{-23}\ \frac{J}{K}$

The observation of quantum interference is limited by an experimenter's ability to isolate the experiment from the environment. This activity investigates how thermal interactions would affect the interference pattern in a hypothetical double-slit experiment. The experimental set-up consists of three distinct components:

- A hot filament emits electrons in all directions.
- A series of collimating filters creates a uniform "pencil" beam of electrons travelling in one direction.
- The electrons in the "pencil" beam gain a small, perpendicular drift velocity, with the average velocity related to the drift temperature, T_{drift}.

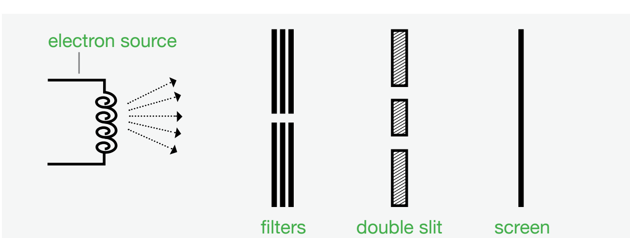


Figure 4.3 Experiment set-up (not to scale)

Figure 4.4

01. The electron source is heated to 7500 K and ejects electrons with a velocity of 5.84 x 10^5 m/s. What is the de Broglie wavelength of the electrons in the beam?
02. The electrons emerge from the collimating filters as a uniform "pencil" beam. They pass through slits that are 200 nm apart and then produce an interference pattern on the detector screen 1.0 m away. Use equation 2 to determine the spacing of the interference maxima, $\Delta x = \Delta x_{interference}$.
03. The electrons in the uniform "pencil" beam will gain a small amount of kinetic energy from unavoidable interactions with the environment expressed by equation 4. For this exercise only the component of the drift velocity that is perpendicular to the original direction of travel (Figure 4.4) will be considered. Use equations 3 and 4 to derive the drift velocity equation:

$$v_{drift} = \sqrt{\frac{k_B\, T_{drift}}{m}}$$

04. The magnitude of the average drift velocity, and resulting drift distance, Δx_{drift}, is dependent on the environmental interactions. Show that

$$\Delta x_{drift} = \sqrt{\frac{k_B\, T_{drift}}{m}} \cdot \frac{L}{v_{beam}}$$

and then calculate the drift distance, Δx_{drift}, if the drift temperature is 0.25 K.
05. Use a sketch to explain how the interference pattern will change if the drift velocity gets too large. Explain why the interference pattern is washed out when

$$\Delta x_{drift} = ½\Delta x_{\text{interference}}$$

06. Prove that the drift temperature that will just wash out the interference pattern is given by:

$$T_{drift} = \left(\frac{h^2}{k_B}\frac{1}{4d^2}\right)\frac{1}{m}$$

07. Calculate the drift temperature that will just wash out the interference pattern using the data from question 02.
08. Consider the relationship derived in question 06. Why does the mass of the object limit the visibility of an interference pattern in a realistic double-slit experiment?

Worksheet 05:
Investigating the Nature of the Electron

In this activity, you will use the double-slit experiment to investigate the nature of classical objects and classical waves and compare them to electrons.

Part 01: Classical Particle Behaviour

EXPERIMENT

In this activity, you will pour sand through two narrow slits, 1.0 cm apart, cut into the bottom of a paper cup. The bottom of the paper cup needs to be approximately 0.5 cm from the paper-covered tabletop.

PREDICTION

(i) Sketch your prediction of how the sand will pile up after it has passed through the slits. Draw the profile of the sand as it would look when viewed from the side. Show the slits in your diagram.

(ii) Provide reasoning for your prediction in one or two sentences.

PROCEDURE

(i) Cut two narrow slits in the bottom of a paper cup 1.0 cm apart.

(ii) Hold the paper cup with the slits in the bottom just above a white piece of paper. Carefully pour sand into the cup and tap it gently to allow some of the sand to pass through the slits (see Figure 1).

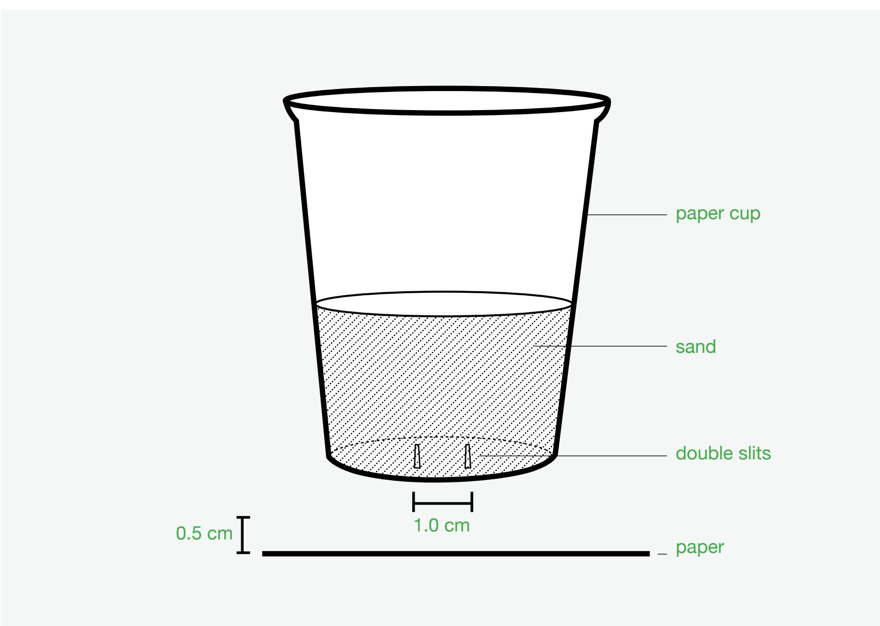


Figure 1 Be sure to keep the bottom of the cup still and very close to the tabletop when pouring the sand through the slits.

OBSERVATIONS AND QUESTIONS

01. Sketch a profile of the pile of sand.

02. Does the profile of the sand match your prediction? Why does the sand form the observed shape?

03. Grains of sand are localized particles. How do two grains of sand interact when they arrive at the same location at the same time?

04. Make a general statement about how classical localized particles behave when passing through a double-slit apparatus and describe the pattern they make after they have passed through the apparatus.

Worksheet 05:
Investigating the Nature of the Electron

Waves behave differently from particles. Before proceeding, recall how waves interact by studying the diagrams below, which show constructive and destructive interference.

Part 02: Classical Wave Behaviour

EXPERIMENT

In this activity, you will model waves passing through two slits using waves drawn on transparencies. You will observe and record how these waves interact when they meet at a screen 15 cm away from the slits.

See Appendix B for wave templates.

PREDICTION

(i) Sketch your prediction of how the two waves will interact when they meet at various locations across the screen. Clearly label regions of complete constructive and destructive interference.

(ii) Provide reasoning for your prediction in one or two sentences.

PROCEDURE

(i) Place wave A on the push pin centered on slit A. Place wave B on the push pin centered on slit B.
(ii) Arrange the transparencies so that they meet at the screen.
(iii) Start at one side of the screen and move along to the other side. Place a mark on the screen indicating the places where total constructive or total destructive interference happens (see Figure 3).

OBSERVATIONS AND QUESTIONS

01. Sketch and label the pattern of constructive and destructive interference.

02. Compare the pattern with your prediction. Explain any discrepancies between your prediction and the experimental observations.

03. Waves are not localized, they are spread out. How do two waves interact when they arrive at the same location at the same time?

04. Make a general statement about how classical waves behave when passing through a double-slit apparatus and describe the pattern they make after passing through the apparatus.

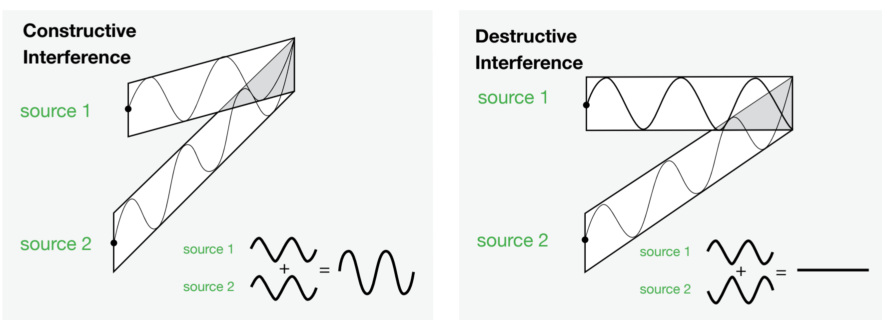


Figure 2 Recall the constructive interference and destructive interference of classical waves.

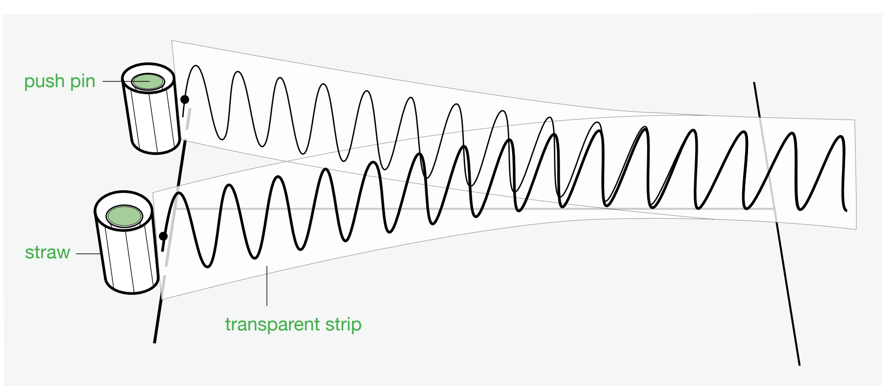


Figure 3 Use waves drawn on transparencies to observe interference.

Worksheet 05:
Investigating the Nature of the Electron

In 1807, Thomas Young published results from a double-slit experiment conducted with light. At the time, his results seemed to solve the debate about whether light was a particle or a wave.

Part 03: Light Behaviour

EXPERIMENT

In this activity, you will shine a laser through two narrowly spaced slits and onto a distant screen.

PREDICTION

(i) Sketch your prediction of how the light will appear on the screen after passing through the two slits.

(ii) Provide reasoning for your prediction in one or two sentences.

PROCEDURE

(i) Carefully shine laser light through two narrowly separated slits and onto a screen as illustrated in Figure 4.

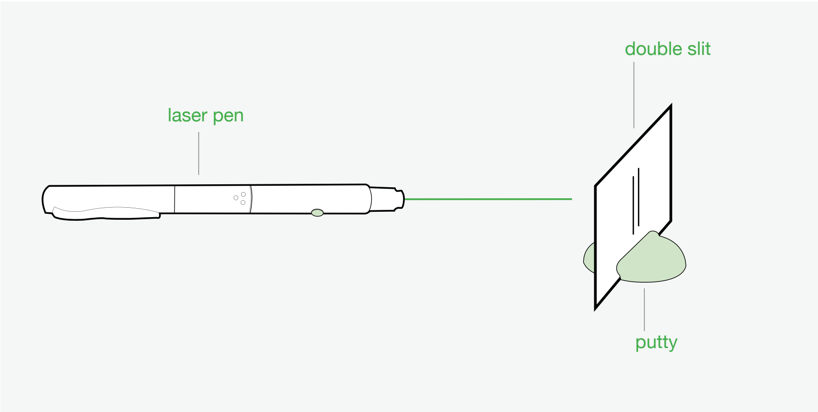


Figure 4 Carefully shine laser light through two narrowly separated slits and onto a screen.

OBSERVATIONS AND QUESTIONS

01. Sketch the resulting image on the screen.

02. Compare your results for light with those for the sand and the waves modelled with transparencies. Based on this comparison, formulate an argument that describes light either as a wave or a particle. Include diagrams.

Worksheet 05:
Investigating the Nature of the Electron

An electron is often described as a particle. This idea can be tested by passing electrons through a double-slit apparatus.

Part 04: Electron Behaviour

EXPERIMENT

In this activity, you will predict how electrons behave when passing through a double-slit apparatus. After making your prediction, you will be provided with data from the actual electron double-slit experiment.

PREDICTION

(i) Sketch your prediction of how the electrons will appear on the screen after passing through the double slits. Assume enough time has elapsed for several thousand individual electrons to pass through the apparatus.

(ii) Provide reasoning for your prediction in one or two sentences.

PROCEDURE

The images in Figure 5 were produced by sending individual electrons through a double slit. Each dot represents an electron striking the detection screen. Carefully analyze the image data to help you answer the questions.

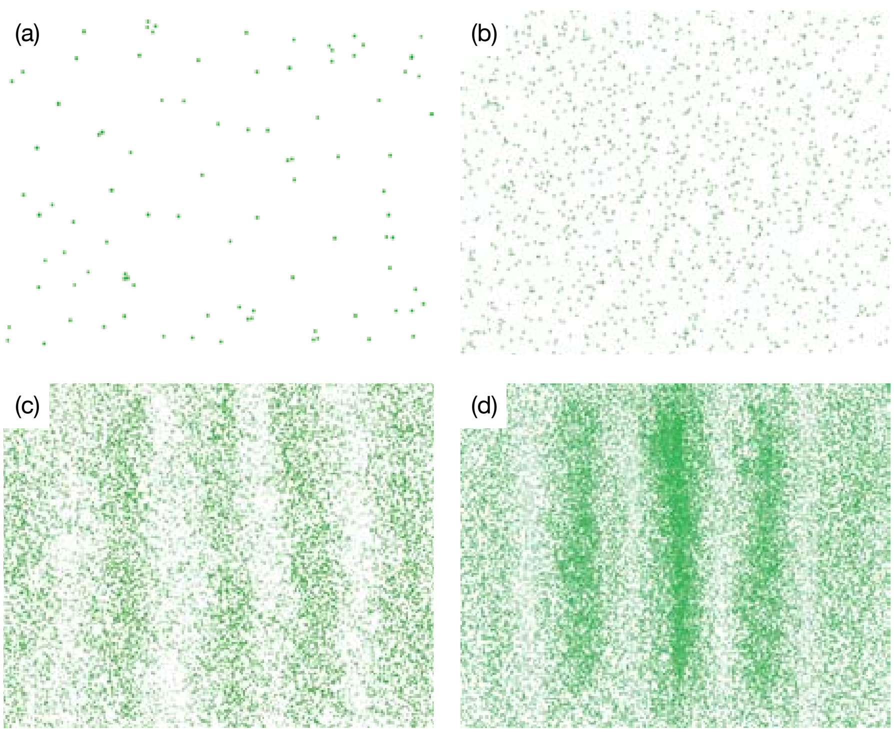


Figure 5 This sequence of images details individual electrons hitting the detection screen at (a) 1 minute, (b) 10 minutes, (c) 25 minutes, and (d) 40 minutes into the experiment.

OBSERVATIONS AND QUESTIONS

01. Consider image (d). How accurate was your prediction? What assumptions did you make when developing your prediction that lead to its relative accuracy or inaccuracy?

02. How does the data support a wave-behaviour description of the electrons?

03. How does the data support a particle-behaviour description of the electrons?

04. Carefully consider the observed electron data. Is it possible to determine whether electrons exhibit strictly wave or particle behaviour? Explain your reasoning as completely as possible with direct reference to the image data and the general statements you made in Part 01 and Part 02 of the worksheet.

Worksheet 05:
Investigating the Nature of the Electron

Part 05: Summary

01. Construct a summary chart for the behaviour of particles and waves in the double-slit experiment. Be as detailed as possible.

02. Use your chart from question 01 to analyze the results for light passing through two slits. According to your summary, is light best described as a particle or a wave? Support your conclusion with a clear statement.

03. Use your chart from question 01 to analyze the results for electrons passing through two slits. According to your summary, are electrons best described as a particle or a wave? Support your conclusion with a clear statement.

04. Are you comfortable with the statement that you have made about the nature of electrons? Does it agree with your current understanding of the electron?

05. Describe what is meant by the phrase, “Electrons exhibit wave–particle duality.”

06. The electron double-slit experiment creates a dilemma for us. How can individual electrons produce an interference pattern? Develop your own explanation for what the electrons are doing as they pass through the apparatus. Summarize your explanation in two or three sentences.

07. The electron double-slit experiment challenges our understanding of nature. In this experiment we have an object acting as both a wave and a particle. Imagine you are going to explain this to your family tonight at dinner. Write a paragraph describing the results of the electron double-slit experiment and your conclusions, using language that they will understand. Be prepared to report back to class tomorrow about the success of your tutorial.

08. This activity provides insight into how difficult it is to describe quantum phenomena using classical ideas about particles and waves. Undoubtedly, the result of the double-slit experiment for single electrons has left you with some intriguing questions. Generate a list of three to five questions about the wave–particle duality of quantum objects. Compare your list with a partner, and be prepared to share some of your questions with the class.

Chapter 01
Classical Background

This chapter of the video:

- *uses the double-slit experiment to review the behaviour of classical particles, water waves, and light.*
- *shows that classical particles are localized. They pass through the slits as individual particles and strike a wall in a familiar and predictable distribution.*
- *shows that water waves are spread out. The slits act as individual sources that produce an interference pattern as the waves overlap.*
- *examines the results of Young's double-slit experiment to support the use of a wave model for light.*

The world of classical physics is relatively straightforward. There is matter and energy, particles and waves. Phenonmena can be described completely as one or the other.

CLASSICAL PARTICLES

In classical physics, matter is made up of particles. The particles are localized, which means their location can be described exactly and they can only be in one place at one time. Localized particles follow trajectories that can be predicted with mathematics using variables such as velocity, acceleration, etc. When two particles are in the same place at the same time they collide and their trajectories change. Careful measurements of a particle's location and trajectory allow us to make very precise predictions about the outcome of any event.

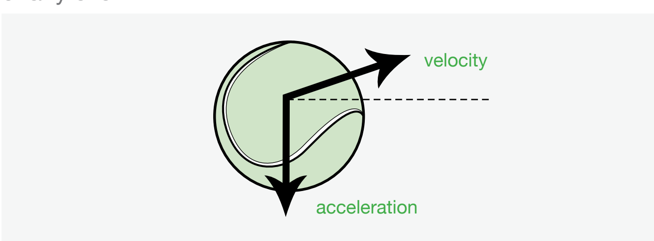


Figure 1.1 This tennis ball can only be in one place at one time. It follows a predictable path.

CLASSICAL WAVES

Energy can be transferred through a medium by the propagation of a wave. A wave is a disturbance that spreads out through the medium by making the particles of the medium move about an equilibrium position. These particles are localized and can only be in one place at one time. When two (or more) waves meet, the medium will add the amplitudes of the waves together and produce a superposition of the waves. Superposition of two (or more) waves can produce an interference pattern and this pattern can be described using geometry.

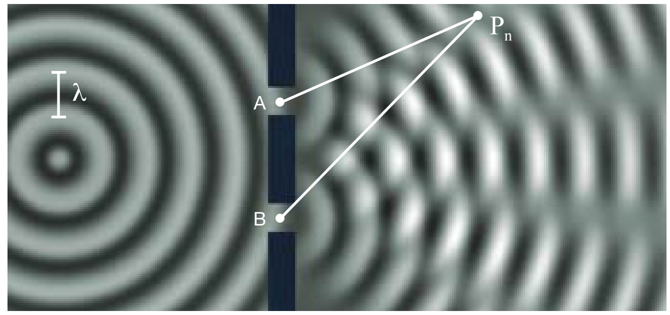


Figure 1.2 When two waves meet they produce an interference pattern.

In Figure 1.2 $$\left|\overline{P_nA} - \overline{P_nB}\right| = (n - \frac{1}{2})\lambda \qquad (1.1)$$

where

$\overline{P_nA}$ = the distance from point P on the *n*th nodal line to source A (m)

$\overline{P_nB}$ = the distance from point P on the *n*th nodal line to source B (m)

n = an integer identifying which nodal line the point is on

λ = wavelength (m)

LIGHT AS A WAVE

In 1803, Thomas Young described the interference of light using several experiments. The experiment that has survived with his name associated with it is the double-slit experiment, in which light shines through two narrow slits. His analysis of the pattern used the geometry of wave interference, and his conclusion was that light must be some sort of wave phenomenon. Young's double-slit experiment seemed to settle the debate about the nature of light in favour of Huygens's wave model.

Figure 1.3 Interference pattern produced by light.

Chapter 02
Wave–Particle Duality with Electrons

This chapter of the video:

- *presents an electron interference experiment using a double-slit.*
- *illustrates how the experiment provides evidence for both the particle nature and the wave nature of electrons.*
- *introduces de Broglie's wave equation for matter.*

WAVE–PARTICLE DUALITY

In classical physics, matter is modelled as a particle. However, in the subatomic world of quantum physics, things are different. The double-slit experiment with electrons highlights the dual nature of subatomic matter, illustrating both particle and wave behaviour, intrinsic to the quantum realm.

Dr. Herman Batelaan and his team at the University of Nebraska-Lincoln have successfully conducted an electron double-slit experiment. Dr. Batelaan's team fired electrons at two tiny slits, only 100 nm wide. Their investigation is the most recent version of an electron double-slit experiment and provides a concrete look into the wave–particle duality of matter on quantum scales

HOW THE EXPERIMENT WORKS

In the experiment, a tungsten filament is heated to a few thousand degrees, causing elections in the filament to be ejected at high speeds. The high-speed electrons pass through narrow apertures that collimate the beam. The beam of electrons is incident on a silicon nitride double-slit barrier. The slits are 100 nm wide and are separated by a distance of 200 nm. After passing through the slits each electron is detected by an electron multiplier that is used to generate a magnified image on a computer monitor. It is impossible to predict where an individual electron will hit the screen. After enough electrons have passed through the apparatus, however, a distinctive interference pattern emerges. Figure 2.3 is a simplified schematic diagram of the experimental set-up.

The intensity of the electron beam can be turned down so that there is only one electron in the apparatus at a time. Surprisingly, despite the fact that electrons are passing through the apparatus one electron at a time, an interference pattern will still develop over time. The interference pattern can be analyzed using the same equations used to investigate Young's double-slit experiment for light.

$$\lambda = \frac{\Delta x \cdot d}{L} \qquad (2.1)$$

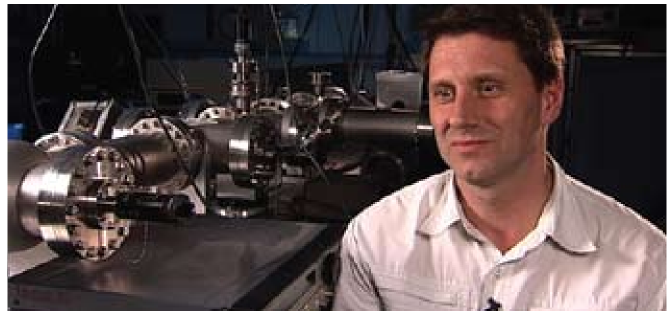


Figure 2.1 Dr. Herman Batelaan in front of the electron double-slit experiment at the University of Nebraska-Lincoln

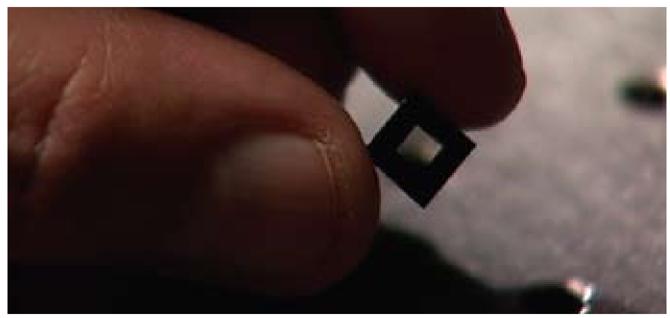


Figure 2.2 The photo shows the actual double-slit barrier used by Batelaan. The slit centres are separated by only 200 nm

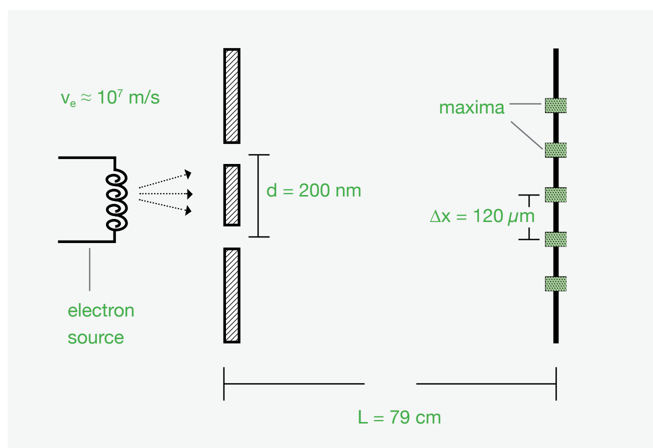


Figure 2.3 Dr. Batelaan's electron double-slit experiment at the University of Nebraska-Lincoln (not to scale)

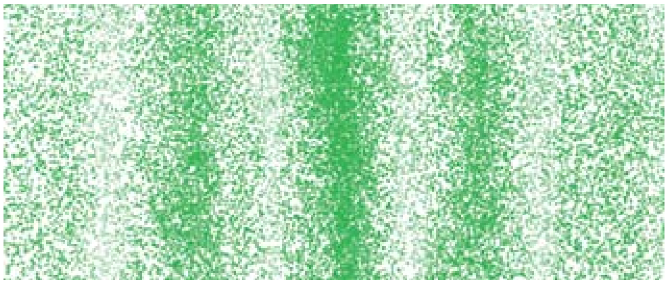


Figure 2.4 An interference pattern is evident after thousands of electrons have been detected

Electron Double-Slit Experiment Data

distance between slits, d	200 nm
width of each slit	100 nm
effective distance from slits to the detection screen, L	79 cm
distance from source to slits	30 cm
temperature of electron source	3000 to 4000 K
electron de Broglie wavelength, λ	$3\text{x}10^{-11}$ m
electron velocity, $v_{electron}$	10^7 m/s
maxima separation distance, Δx	120 μm

where λ = wavelength of electron (m)
Δx = distance between adjacent maxima or minima (m)
L = distance between the slits and the screen (m)
d = slit separation (m)

The interference pattern result raises deep questions about what the electron is actually doing as it travels through the double-slit apparatus, and how seemingly particle-like objects are able to produce an interference pattern. The mathematical formalism of quantum mechanics predicts the interference, but it does not answer any questions about what a specific electron is actually doing inside the apparatus. This ambiguity is what leads to the various interpretations presented in Chapter 5.

MATTER EXHIBITS WAVE PROPERTIES

The wave–particle duality of an electron was part of de Broglie's 1924 doctoral thesis, in which he derived his matter–wave equation (see equation 2.2). Interestingly, an electron interference experiment was not actually conducted until 1961 when Claus Jönsson of Tübingen, Germany finally verified the 1920s theoretical predictions. By then, the result was not at all surprising and received little fanfare.

The wave nature of matter is mathematically expressed by the de Broglie equation

$$\lambda = \frac{h}{mv} \qquad (2.2)$$

where λ = wavelength (m)
h = Planck's constant (6.626×10^{-34} J·s)
m = mass (kg)
v = velocity (m/s)

The variables contained in the de Broglie equation help illustrate the wave–particle duality of matter. The object's wavelength, a wave property, is determined from the object's mass and velocity (typically associated with a particle) and Planck's constant. Planck's constant is a common feature in equations dealing with quantum physics. The constant is extremely small and is related to the minimum size of the discrete units of energy, mass, spin, and other quantum descriptors.

It is important to emphasize that any quantum object exhibiting wave–particle duality only ever demonstrates one behaviour at a time. For example, in the double-slit experiment with electrons, the interference pattern is built up one electron at time and it is this pattern that provides the evidence for wave-like behaviour. However, the individual electrons that are emitted and strike the screen at localized spots provide evidence for particle-like behaviour. This dual nature is not observed in the macroscopic world, and it highlights a key difference between descriptions in classical physics and quantum physics. A classical particle always behaves as a particle, and never requires a classical wave model to describe its behaviour. A quantum object is not a classical particle or a classical wave. Careful use of language is required to correctly describe a quantum object. Phrases that describe the observed behaviour are preferred over statements about what a quantum object actually is. For example, it is safer to say that an electron "behaves like a particle" than an electron "is a particle." Interpretations about what an electron "is" are discussed in the Chapter 5 Summary.

THE WAVEFUNCTION – A MATHEMATICAL DESCRIPTION

Quantum physics mathematically addresses wave–particle duality and the behaviour of an electron by using a mathematical wave called a wavefunction. A wavefunction gives the probabilities for finding an electron at all of the possible locations that it can be observed. If the amplitude of an electron's wavefunction at a particular location is large, there is a high probability of finding the electron there. If the amplitude is small, there is a low probability of finding the electron there. The wavefunction is a mathematical description and, in the absence of a specific interpretation, does not answer the question about what an electron is.

Chapter 03
Wave – Particle Duality with Light

This chapter of the video:

- *shows how light, which had previously been modelled as a wave, also demonstrates wave–particle duality.*
- *introduces the formula for the energy of a photon.*
- *illustrates how this strange behaviour is also seen in protons, neutrons, atoms, and even very large molecules (buckyballs).*
- *presents the differing opinions of researchers on how big a quantum object can be.*

EVIDENCE FOR PHOTONS

The nineteenth century opened with the publication of Young's double-slit experiment, which firmly established the wave nature of light. As the century drew to a close there were several experiments that pointed to the need for a different model. By 1905, Einstein was using a particle model for light in his explanation of the photoelectric effect. The double-slit experiment showed that light came in packets whose energy could be calculated by the equation

$$E = \mathrm{h}f \qquad (3.1)$$

where E = the energy of the photon (J)
h = Planck's constant (6.63 x 10^{-34} J·s)
f = the frequency of the photon (Hz)

Note how the variables in this equation illustrate the wave–particle duality of light. A photon, which is detected as a localized object like a particle, has an energy that is proportional to its frequency, which is a wave-like property. The two aspects are connected by Planck's constant, the same constant that is in the de Broglie wavelength equation, $\lambda = h/p$. The de Broglie equation holds for all quantum objects, including photons, and therefore a photon has momentum even though it has no mass.

A demonstration of Young's double-slit experiment with individual photons was done by Geoffrey Ingram Taylor in 1909. He used extremely faint light, so that there was only one photon in the apparatus at a time. The light was so faint that it required a three-month exposure time before the many individual photons were able to form an interference pattern.

WAVE–PARTICLE DUALITY AND LIGHT

All of the wave-like properties of light can be demonstrated using individual photons. This means that all of the demonstrations and experiments that we do with light have quantum physics at their core. For example, thin-film interference seems quite reasonable as a wave phenomenon; part of the wave reflects from the top surface of the thin film and part reflects from the bottom. These two parts interfere with each other to produce maxima and minima that vary with the thickness of the film. But this interference pattern forms even for only one photon at a time. How can a single photon do this? It is the same counter intuitive result that is found in the double-slit experiment.

The model for light scientists use often depends on the energy of the radiation with which they are working. Individual photons are easiest to detect if they are high energy, and so physicists who work with low-energy radio waves rarely consider light's particle-like behaviour, and physicists who deal with high-energy gamma radiation rarely see the wave-like behaviour.

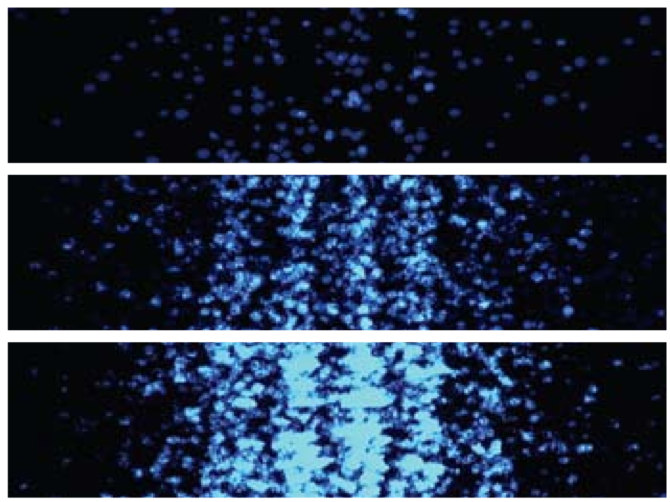


Figure 3.1 These photos show how an interference pattern forms from many individual photons.

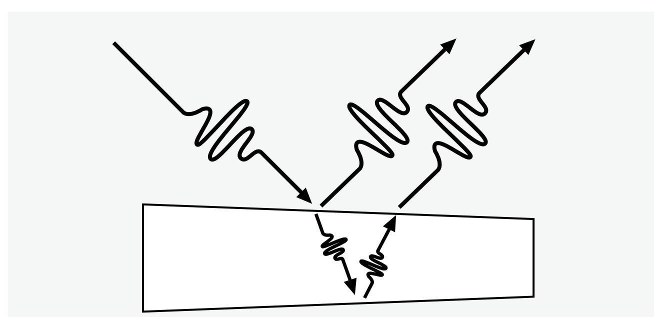


Figure 3.2 Thin film interference can be observed even when experimenters only use one photon at a time.

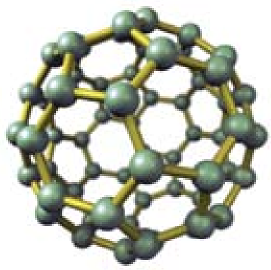


Figure 3.3 *Buckminsterfullerene or buckyballs consist of 60 carbon atoms*

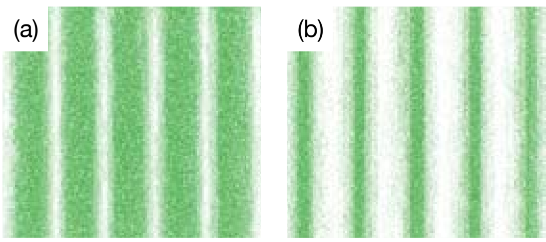


Figure 3.4 *Interference pattern produced by two slits (a) versus many slits (b)*

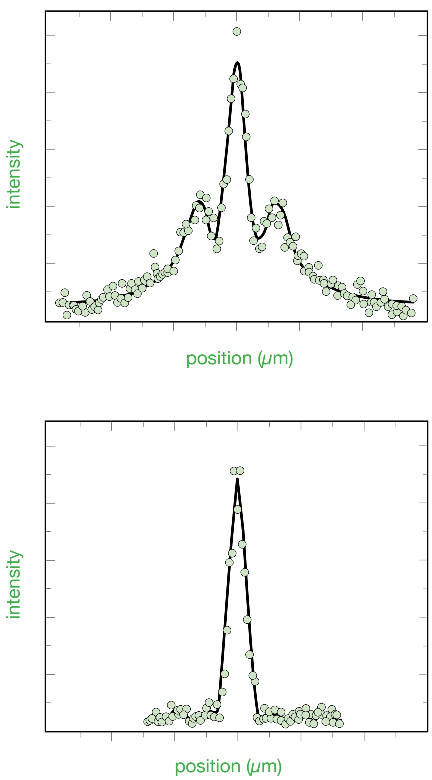


Figure 3.5 *The results of the buckyball experiment.The graphs show the results without (top) and with a diffraction grating (bottom).*

THE QUANTUM NATURE OF LARGE OBJECTS

Larger objects should also have de Broglie wavelengths, but these are much harder to demonstrate. Their wavelengths are smaller because their mass is greater. Recall the de Broglie equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (3.2)$$

For example, buckminsterfullerene, or buckyballs, are made of 60 carbon atoms, so each one is about 600 000 times more massive than an electron. In order to make their wavelengths large enough to be detected, physicists had them travel much more slowly—200 m/s rather than 120 000 000 m/s. Even so, the wavelength of the balls was only 0.0025 nm, which is 400 times smaller than the 1 nm size of the molecule itself!

It gets more difficult to demonstrate interference as the wavelengths get smaller. Separation between the maxima is given by

$$\Delta x = \frac{\lambda L}{d} \qquad (3.3)$$

We can compensate somewhat for the tiny wavelength by using a very small slit separation, *d*. The slits in the buckyball experiment were 50 nm wide and separated by 100 nm. To make the separation of maxima even clearer, physicists used a diffraction grating with many slits, rather than just two. This does not change the separation between maxima, but it does make the maxima more concentrated and the minima more spread out (see Figure 3.4).

The results of the buckyball experiment are shown in Figure 3.5. The graphs show the results without a diffraction grating (top) and with (bottom). Note that there are only two interference maxima produced beyond the central one and these are really not all that clear. This shows how difficult it is to demonstrate the interference of such a “large” object. The same physicists have also shown interference with a fluorinated buckyball made of 60 carbon atoms plus 70 fluorine atoms, and they are trying for larger molecules. The physicists in the video disagree as to whether there is a theoretical limit or just a practical, technological limit to showing quantum effects with large objects. The answer is not known.

Chapter 04
Measurement Disturbance

This chapter of the video:

- *describes how detectors placed next to each slit reveal that half of the electrons went through each slit.*
- *describes how the act of measuring the electrons at the slits causes the interference pattern to disappear.*
- *outlines how researchers in Tübingen, Germany have verified these measurement disturbance results.*

In the double-slit experiment with tennis balls, each ball passes through just one slit and no interference pattern is observed. With water, the wave passes through both slits and an interference pattern is observed. An interference pattern is also observed with electrons. This surprising result raises a question about how each electron passes through the slits. The answer is not obvious. The fact that we always observe electrons as localized particles suggests that each electron goes through just one slit, like a tennis ball. However, if that was true, electrons would form the same distribution that the tennis balls make. Instead, they form an interference pattern. Does this mean that each electron somehow goes through both slits, like a wave?

To find out exactly how electrons pass through the slits, we can place detectors next to each slit. Physicists in Tübingen, Germany did just this in 2002. Their detector consisted of a slab of silicon placed near both slits. When a (negative) electron went through one of the slits it attracted positive charges in the silicon creating an electrical current, which caused the silicon to heat up. From the heating data, the Tübingen scientists were able to determine that an equal number of electrons went through each slit.

The electrons significantly above the silicon slab did not interact and were not measured, shown by the intact interference pattern at the top of Figure 4.1. However, electrons passing near the bottom of the slits were measured, and the interference pattern was destroyed and replaced by a completely random distribution of hits, shown at the bottom of Figure 4.1. The act of measuring the electrons had disturbed them and the pattern they produced on the screen. This phenomenon is called *measurement disturbance*. It is one of the defining features of quantum physics. In classical physics, we can measure an object without affecting it. For example, we can measure the speed of a car with a radar gun without altering the car's speed in any significant way. However, if we measure an electron, or any other quantum object, we change its behaviour in a significant way.

WHY DOES MEASUREMENT DISTURB?

To measure which slit an electron goes through, we have to physically interact with it. In the Tübingen experiment, electrical charges in the silicon "detector" exerted electromagnetic forces on the electrons. The interaction with each electron was largest when the electron passed close to the silicon, and it weakened as the distance increased. For example, if we measure which slit electrons go through by shining light on them, photons hit the electrons and bounce off them. Interactions like these have an effect on electrons. When a photon hits an electron, the electron rebounds and changes its direction of motion. As photons collide with each electron in a slightly different way, different electrons travel off in different directions. As a result, they hit the screen all over the place, destroying the interference pattern.

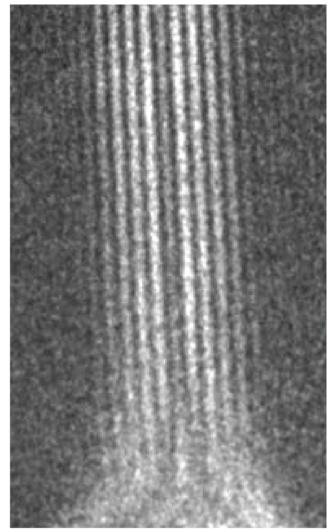


Figure 4.1 Measurement disturbance data from Tubingen. Notice how the clear interference pattern near the top is destroyed near the bottom due to the detector.

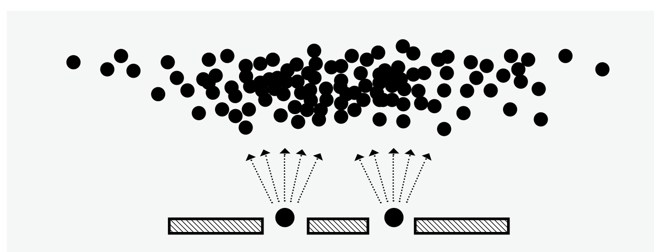


Figure 4.2 Electrons that have been detected by photons move in va wide range of directions due to their interaction with the photons.

HEISENBERG'S UNCERTAINTY PRINCIPLE

The concept of measurement disturbance is closely related to Heisenberg's uncertainty principle. This principle says there is a fundamental limit on how accurately we can make simultaneous measurements of the position and momentum of a quantum object. So, if we know the position of a quantum object with great accuracy, then we know very little about its momentum. When a detector measures which slit an electron passes through, we know its position with great accuracy. So Heisenberg's uncertainty principle says that its momentum is highly uncertain. This means that the electron could be moving in one of a wide range of directions, as shown in Figure 4.2. This leads to electrons hitting the screen all over the place and to the pattern at the bottom of Figure 4.1.

Chapter 05
Interpretations and Applications

This chapter of the video:

- *presents four different perspectives currently used to try to make sense of the quantum reality (Collapse, Pilot Wave, Many Worlds, and Copenhagen interpretations).*
- *highlights the surprising variety of technologies that owe their existence to the quantum nature of our world.*

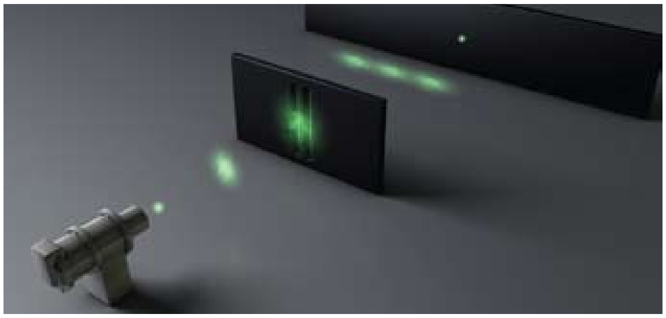
WHERE EXPERIMENTAL FACTS END AND INTERPRETATION BEGINS

The double-slit experiment demonstrates the way nature really behaves. The electron's dual wave and particle behaviour is a fact, as strange as it seems. When we use a measurement device and look at an electron to see what it is doing, we perturb it and actually change what happens. That leads to an understanding of nature at the quantum level which is very different from the familiar models of classical physics.

In the double-slit experiment, electrons are detected as particles at the screen, but while passing through the slits their behaviour seems to be governed by waves. Nobody really knows what the electrons are doing between the source and the detector. We have equations that make very accurate predictions about the results of the double-slit experiment, but quantum physics does not seem to answer the question about what is actually going on between the source and the detector. In the absence of a clear answer, physicists have developed various interpretations to complete the picture and describe what might be happening in the quantum world.

COLLAPSE INTERPRETATION

Scientists who subscribe to the Collapse interpretation make a choice. They believe that when you accept the electron's wave nature, you must give up on the electron's particle nature.



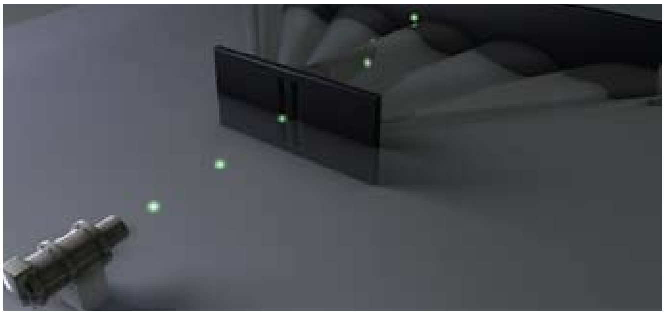
In this interpretation, the electron leaves the source as a particle that is governed by one set of laws, but then "expands" into a spread-out wave as it passes through the slits. The electron is now governed by new laws.

However, before we can measure this wavy, spread-out quantum electron it "collapses" back into a particle and arrives at only one of the many possible places on the screen.

The consequence of choosing the Collapse interpretation line of thinking is that you must accept that an electron physically changes from particle to wave and back again. These two realities, including the laws that describe them, alternate uncontrollably.

PILOT WAVE INTERPRETATION

The Pilot Wave interpretation avoids this unexplained collapse altogether. Scientists who subscribe to this interpretation choose to believe that the electron always exists as a classical particle and is only ever governed by one kind of physical law, for both the familiar classical as well as quantum phenomena. However, to account for the electron's wave behaviour this description requires the introduction of an invisible guiding wave.



In this interpretation, wave-particle duality is explained by assuming that electrons are real particles all of the time, and are guided by an invisible wave. The electron's wave nature is attributed to this abstract wave, called a Pilot Wave, which tells the electron how to move. To obtain the interference pattern in the double-slit experiment, this wave must be everywhere and know about everything in the universe, including what conditions will exist in the future. For example, it knows if one or two slits are open, or if a detector is hiding behind the slits.

The Pilot Wave interpretation embodies all of the quantum behaviour, including all the interactions between classical objects like the electron, the two-slit barrier, and the measuring devices. In contrast to the Collapse interpretation where the collapsing electron wave was considered real, in the Pilot Wave interpretation the wave is an abstract mathematical tool. This interpretation has a consequence. The Pilot Wave interpretation, which was invented to deal with an electron as a real physical object, suffers the fate of being permanently beyond detection.

MANY WORLDS INTERPRETATION

Supporters of the Many Worlds interpretation, similar to the Pilot Wave idea, choose to accept that electrons are classical particles. Then they go even further, demanding that all elements of the theory must correspond to real objects—unlike the collapsing electron or the Pilot Wave. Supporters insist on only measurable, physical objects within the world. This world is constantly splitting into many copies of itself.

When electrons demonstrate wave behaviour they exist in a superposition of many different states. To Many Worlds supporters, who maintain the idea of an electron as a classical particle, a parallel universe must exist for each of the electron's possible states. When the electron reaches the slits, it has to choose which slit to go through. At that moment, the entire universe splits into two. In one universe, the electron passes through the left slit as a real particle. In the other universe it passes through the right slit as a real particle. The consequence of accepting the Many Worlds interpretation, with many quantum particles constantly facing similar choices, is the requirement that our universe must be constantly splitting into an almost infinite number of parallel universes, each having its own copy of every one of us.

COPENHAGEN INTERPRETATION

Advocates of the Copenhagen interpretation choose to limit their discussion directly to the experiment and to the measurements on physical objects. Questions are restricted to what can be seen and to what we actually do. They try to think about experiments in a very honest way, without invoking extra theoretical ideas like the on-off switching of the Collapse idea, or the guidance supplied by the invisible Pilot Wave, or the proposed splitting into Many Worlds.

It is tempting to come up with mental pictures about what is happening that go beyond the results of an experiment, and to try to interpret what is happening by means of those hidden theoretical mechanisms. The previous interpretations attributed the mysterious wave–particle duality to imaginative mathematics. In the Copenhagen interpretation much of this mystery is attributed to what happens when an experimenter enters the lab and interacts with the quantum mechanical system. With the Copenhagen perspective, the mathematics only deals with the experimenter's information about measurement interactions with the quantum mechanical system.

The consequence of accepting the Copenhagen interpretation is a fundamental restriction on how much you can read into experimental results. We know that electrons

Table 6.1 A summary of the choices physicists make and the resulting unsettling feature for each interpretation associated with wave–particle duality.

Interpretations	Assumptions that physicists choose to believe about reality			Unsettling feature
	One set of laws governs electrons.	Only one universe exists.	All objects are real.	
Collapse	X	✓	✓	Random switching between particles and waves, and between classical and quantum laws
Pilot Wave	✓	✓	X	Invisible, undetectable guiding wave that exists in a purely abstract mathematical space
Many Worlds	✓	X	✓	Infinite number of copies of the universe
Copenhagen	?	?	?	Accept that some questions cannot be asked
	Physicists choose to believe that descriptions of reality must be restricted to the measurements that they take.			

are particles when they are fired from the source, and we know that they are particles when they hit the screen. What happens to electrons in the middle, what they are "doing", or what they really "are" is not possible to know. In the Copenhagen interpretation these are unfounded questions. We may call an electron a wave or a particle, but ultimately those names are no more than suitable models.

Although the discussion about an adequate understanding of quantum physics is still unsettled, it is important to realize that all of the interpretations predict the observed experimental results. Leaving the unanswered questions about the foundations of the quantum universe temporarily aside, many physicists have gone on to put quantum physics to use. Engineers use quantum physics to make predictions about experiments, to construct devices, and to explore new technological applications.

APPLICATIONS OF WAVE–PARTICLE DUALITY

Quantum physics has revolutionized society with applications such as lasers, LEDs, and solar cells. Microelectronics led to computers, the Internet, and the Information Age. The electron microscope has opened the door to nanotechnologies. The next generation of innovators are exploring quantum cryptography, quantum computing, and many more possibilities.

ELECTRON MICROSCOPE: ELECTRON WAVES Resolution is the ability to form a clear image and is related to the wavelength of the incident radiation. A light microscope can resolve objects as small as 2×10^{-7} m, which is about half the wavelength of violet light. The de Broglie wavelength of electrons can be a thousand times smaller than violet light. This dramatic reduction in wavelength allows an electron microscope to resolve objects that are incredibly tiny.

The electron beam is focused by electric and magnetic fields instead of glass lenses. Eli Burton and his students built the first practical electron microscope at the University of Toronto in 1938. Today there are transmission (TEM), scanning (SEM), and scanning tunnelling (STM) electron microscopes. Electron microscopy provides visually stunning examples of how engineers are able to use the wave nature of electrons.

Figure 5.1 In this electron micrograph the flea's eye (in red) and mandibles used to suck the host's blood (long tubes) are clearly visible.

ELECTRONICS: ELECTRON WAVES The transistor is at the heart of every electronic device. A typical computer chip holds 200 million transistors. Engineers use a wave model of electrons when they design and build transistors. Every time you use a cell phone, MP3 player, computer, or anything electronic you are taking advantage of the quantum nature of electrons. Quantum computers that will exploit nature's fundamental quantum properties offer untold promise for the future. The transistor, which was once the size of an apple and confined to fundamental research, spawned the Information Age and is now woven into the very fabric of society. We can only imagine what quantum-based innovations will bring.

PHOTONS: PARTICLES OF LIGHT A particle of light, known as a photon, is at the heart of the light detectors needed for remote control systems, digital cameras, and solar cells. Running a solar cell process in reverse yields light emitting diodes (LEDs).

Positron Emission Tomography (PET) is a powerful medical imaging system. The patient swallows a small amount of positron emitting material, and then whenever a positron and electron collide, they annihilate and emit two photons. Detectors capture these photons to produce detailed and dynamic images of biochemical processes within our body.

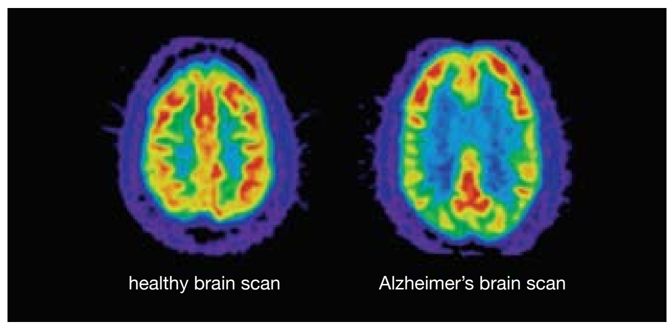


Figure 5.2 This Positron Emission Tomography (PET) scan shows maximum, healthy blood flow in red and limited blood flow in blue.

FUTURE APPLICATIONS

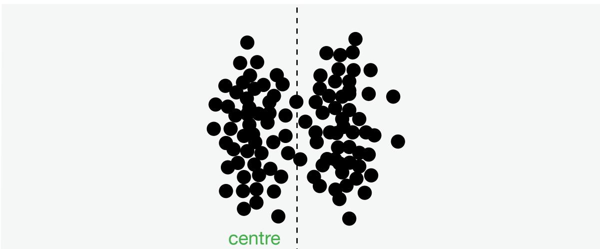
Individual photon detection is also essential for quantum cryptography, which ensures secure data transmission using fundamental principles of quantum physics. If someone tries to listen in, they necessarily disturb the system, just as electron detection at the slits destroys the double-slit interference pattern.

Today's supercomputers are number crunching behemoths, but may fade to insignificance if the promise of quantum computers comes true. Quantum computing and all of its related endeavours are currently exciting and active areas of research around the globe.

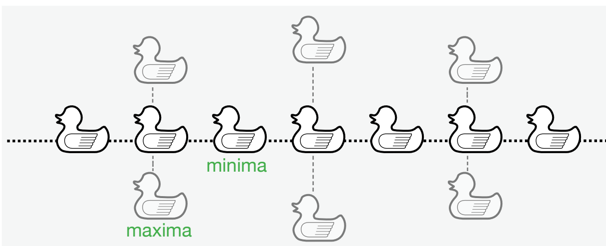
Worksheet Solutions

Worksheet 01: Video Summary

01.



02. (a)



(b) The pattern resembles the bright and dark lines of an interference pattern for light generated using a double-slit apparatus. The length of a line represents the energy at that point. Light mimics this distribution both in terms of alternating maxima and minima and the fact that the central maxima is always brightest (most energy) with subsequent maxima getting dimmer and dimmer (less energy).

03. (a) The distance between maxima is related to the wavelength of the interfering waves and to the slit separation. Water waves in a lab have wavelengths on the order of centimetres. The small separation between electron maxima suggests that the electron's de Broglie wavelength must be very small.
(b) The electrons are detected at the screen as localized particles.
(c) The interference pattern suggests wave behaviour.
(d) Answers will vary. One important point to clarify with students is that the particle nature and the wave nature never occur simultaneously. The wave nature is inferred by the creation of an interference pattern after several thousand electrons have been sent through the apparatus. The particle nature is observed every time an electron is detected, and it is worth noting that only whole electrons have ever been detected.

04. (a) Use the wave equation to determine the frequency:

$$c = \lambda f$$

$$f = (3.0\times10^{8}\ \tfrac{\text{m}}{\text{s}}) \Big/ (5.8\times10^{-7}\ \text{nm}) = 5.17\times10^{14}\ \text{Hz}$$

$$f = 5.2\times10^{14}\ \text{Hz}$$

Use the equation for the energy of a photon to determine the energy:

$$E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(5.17\times10^{14}\ \text{Hz})$$

$$E = 3.4\times10^{-19}\ \text{J}$$

(b) Individual photons are detected at the screen at localized spots.
(c) The formation of an interference pattern.

05. Use the de Broglie wavelength equation:

$$\lambda = h/(mv)$$

$$v = h/(m\lambda)$$

$$v = (6.626\times10^{-34}\ \text{J}\cdot\text{s}) \Big/ (1.7\times10^{-24}\ \text{kg})(2.8\times10^{-12}\ \text{m})$$

$$v = 140\ \tfrac{\text{m}}{\text{s}}$$

06. The act of measuring which slit an electron passes through destroys the interference pattern. The researchers suggest that the interaction between an electron and a measuring photon jostles the electron in such a way as to destroy the interference pattern.

07. There is no consensus about what is happening in between the source and the screen. This lack of consensus has led to several interpretations about what an electron actually is and what is happening between the source and the screen. In the video four interpretations are discussed: Collapse, Pilot Wave, Many Worlds, and Copenhagen. The Copenhagen approach is only loosely called an interpretation in that its line of thinking is restricted to the observable data, and no questions about what is happening between the source and the detector are considered.

08. Technologies based on quantum physics and discussed in the video include: electron microscopes, lasers, remote controls, traffic lights, computers, and any electronic device.

Worksheet 02: Concept Questions

01. (b) The pattern results from the sum of the contributions made by each slit.

02. (c) Constructive interference: when a resultant wave has a larger amplitude than either of its component waves. Destructive interference: when a resultant wave has a smaller amplitude than either of its component waves.

03. (c) A single wave passing through two slits will produce an interference pattern with a central maxima as depicted in (c). Option (d) is incorrect because it shows a central maxima that is twice as wide as adjacent maxima. Option (b) has a central minima, which is only possible if the two waves passing through the slits are exactly out of phase with each other. This is not possible when a single wave is incident on the slits.

04. (c) Classical particles are localized—they can be found in one particular location and they do not interfere. When two classical particles interact they do not form a single larger particle and they do not cancel each other out and disappear. Classical particles form a distribution on the screen that is the sum of contributions made by each slit.

05. (c) Different colours have different wavelengths and will have their maxima and minima at different positions. This produces bands of varying colours. However, in the centre all wavelengths will constructively interfere to produce a white central maxima.

06. (c) This question addresses a common line of reasoning provided by students in an attempt to explain the electron interference pattern. Students often suggest that the interference pattern appears after enough electrons have been fired at the double-slit apparatus because of the interactions of electrons with other electrons as they move through the device. They reason that water is made of many water molecules which exert forces on each other. This interaction between water molecules forms the waves and interference patterns, so it is reasonable to assume that the electrons could be doing something similar. To eliminate the possibility of electrons interacting they have to be sent in one by one. They still produce an identical interference pattern—something water would not do. (A student who is really paying attention may point out that a water molecule, like an atom or buckyball, has a de Broglie wavelength. This means that you could get an interference pattern with many individual water molecules. However, it would be much, much smaller than the original pattern. The wavelength of a water molecule is on the order of nanometres, not centimetres.)

07. (a) This question should provoke students to think about what evidence supports an electron's wave-like properties and what evidence supports particle-like properties. Electrons are only ever detected as single localized objects; partial electrons have never been detected. The electron's wave nature is only inferred from the interference pattern produced over time. These dual behaviours do not permit scientists to say electrons are particles or waves. What can be said is that electrons behave like waves and they behave like particles.

08. (b) This question investigates the energy of a photon, $E = hf$. A UV photon has a higher frequency than a visible-light photon, and therefore has sufficient energy to trigger the chemical reactions that cause sunburn. This question also provides an opportunity to learn that photons have no mass even though they have momentum ($p = h/\lambda$), and that they all travel at the same speed (c). Furthermore, the wavelength ($\lambda = c/f$) is smaller for UV light.

09. (c) This question encourages students to think about the limits of quantum physics. It may turn out that the de Broglie wavelength equation does not hold for larger objects; however, there is as yet no evidence for a quantum limit. A tennis ball moving at normal speeds will have a de Broglie wavelength that is much, much smaller than an atom. Have students do an order of magnitude calculation. The tennis ball wavelength will be around 10^{-35} m. That is 10^{25} times smaller than a typical atom. The only way to increase the wavelength significantly is to slow it down. If you slowed it down enough that it would travel one metre in the amount of time that the universe has existed, you will still only increase the wavelength by 10^{17}, leaving a wavelength of 10^{-8} m.

10. (a) This question shows that both the wave and particle nature of quantum objects can be observed in the context of the double-slit experiment. The arrival at specific locations is particle-like behaviour and the interference pattern is wave-like behaviour. Each individual quantum object shows evidence of both types of behaviour.

11. (c) This question forces students to think about the fact that electrons have only ever been observed as whole, intact, localized particles. Electrons are only found as complete electrons, with a standard charge and mass, in one specific place. However, if we know which slit the electron went through, the interference pattern disappears.

12. (d) This question will help students identify the difference between experimental evidence and attempts at explanations based on conjecture. Physicists know statistically where electrons are likely to hit, but they cannot say where a specific electron will hit. If scientists measure to see which slit an electron went through, the interference disappears. They know that

two slits are necessary for an interference pattern, but that does not necessarily mean that an electron went through both, and a split electron has never been found. Fundamentally, we do not know what the electron does between the source and the screen. Physicists have many currently untestable ideas about what might be happening, called interpretations. It is important to emphasize that although the interpretations differ about what is happening within an experiment, the formal mathematical predictions of quantum physics predict exactly what is experimentally observed.

13. (b) In the Pilot Wave interpretation, the Pilot Wave is aware of all possible paths and guides a particle-like electron through one or the other slit. In the Many Worlds interpretation, the universe in which the electron goes through the left slit branches off from the universe in which the electron goes through the right slit. However, in any given universe the electron only goes through one slit. In the Collapse interpretation an electron wave goes through both slits. The Copenhagen interpretation restricts thinking to physically observable phenomena and does not allow for questions that do not have a measurable answer.

14. (d) Electrons and photons do not have well-defined sizes—just wavelengths. Electrons are used in electron microscopes because their smaller wavelengths can resolve smaller objects.

15. Answers will vary. This question should generate lively discussion. Solar panels provide electrical power to remote regions without the need for large-scale infrastructure. Lasers are used in DVD players, bar-code scanners, and specialized medical and industrial equipment. Transistors are the heart of all computers and are necessary for cell phones, iPods, medical imaging, and most household appliances.

Worksheet 03:
Mathematical Investigation of Wave-Particle Duality

01. (a) From the question, d = 200 nm and L = 0.790 m. From the figure (the image was magnified 100X, so the measurements must be adjusted by 100):

$$3\Delta x = \frac{3.60\,\text{cm}}{100} \quad \text{therefore} \quad \Delta x = 1.20\times10^{-4}\ \text{m}$$

$$\lambda = \frac{\Delta x d}{L} = \frac{(1.20\times10^{-4}\ \text{m})(2.00\times10^{-7}\ \text{m})}{0.790\,\text{m}}$$

$$\lambda = 3.04\times10^{-11}\ \text{m}$$

(b) From part (a) λ = 3.04 x 10^{-11} m, and from known data $m_{electron}$ = 9.11 x 10^{-31} kg.

$$p = \frac{h}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{3.04\times10^{-11}\ \text{m}} \qquad v = \frac{p}{m} = \frac{2.181\times10^{-23}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{9.11\times10^{-31}\ \text{kg}}$$

$$p = 2.181\times10^{-23}\ \frac{\text{J}\cdot\text{s}}{\text{m}} \qquad v = 2.394\times10^{7}\ \frac{\text{m}}{\text{s}}$$

$$p = 2.18\times10^{-23}\ \frac{\text{J}\cdot\text{s}}{\text{m}} \qquad v = 2.39\times10^{7}\ \frac{\text{m}}{\text{s}}$$

(c) From part (b) v = 2.394 x 10^{7} m/s, and from known data $q_{electron}$ = 1.602 x 10^{-19} C. The energy of the electric field provides the kinetic energy of the electron:

$$qV = \tfrac{1}{2}mv^2$$

$$V = \frac{mv^2}{2q} = \frac{(9.11\times10^{-31}\ \text{kg})(2.394\times10^{7}\ \frac{\text{m}}{\text{s}})^2}{2(1.602\times10^{-19}\ \text{C})}$$

$$V = 1.63\times10^{3}\ \text{V}$$

02. (a) From the question V = 30 x 10^{3} V, and from known data $q_{electron}$ = 1.602 x 10^{-19} C, $m_{electron}$ = 9.11 x 10^{-31}kg. The kinetic energy of the electron is equal to the energy of the electric field:

$$\tfrac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(3.0\times10^{4}\ \text{V})}{9.11\times10^{-31}\text{kg}}}$$

$$= 1.03\times10^{8}\ \frac{\text{m}}{\text{s}}$$

$$v = 1.0\times10^{8}\ \frac{\text{m}}{\text{s}}$$

Note: at this speed we would normally take relativity into account. For simplicity, we will restrict ourselves to non-relativistic calculations.

(b) From part (a) v = 1.03 x 10^{8} m/s. From known data $m_{electron}$ = 9.11 x 10^{-31}kg and h = 6.626 x 10^{-34} J·s

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(1.03\times10^{8}\ \frac{\text{m}}{\text{s}})}$$

$$\lambda = 7.1\times10^{-12}\ \text{m}$$

(c) $\lambda_{green} = 550\,\text{nm} = 5.50\times10^{-7}\text{ m}$

$\lambda_{electron} = 7.1\times10^{-12}\text{ m}$

$$\frac{\lambda_{green}}{\lambda_{electron}} = \frac{5.50\times10^{-7}\text{ m}}{7.1\times10^{-12}\text{ m}} = 7.8\times10^{4}$$

Therefore, the wavelength of green light is over 70 thousand times larger!

(d) In the question we are given the maximum resolution for an optical microscope. If we use a simple ratio we find a resolution for the TEM:

$RP_{optical} = 200\text{ nm}$

$$RP_{electron} \cong \frac{200\text{ nm}}{7.8\times10^{4}} \cong 2.6\times10^{-3}\text{nm}$$

(e) The actual resolving power for a typical TEM is about 0.2 nm, which is about 1000 times better than an optical microscope. Other limiting factors involve complex issues surrounding the beam width, spherical aberration, and technical limits. TEMs must also take relativity into account because of the extremely high electron speeds.

03. (a) The power rating describes how much energy per second leaves the laser. The wavelength tells us how much energy per photon.

$P = 1.0\,\text{mW} = 1.0\times10^{-3}\,\frac{\text{J}}{\text{s}}$

$\lambda = 6.33\text{ x }10^{-7}\text{ m}$

$$E = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.0\times10^{8}\,\frac{\text{m}}{\text{s}})}{6.33\times10^{-7}\text{ m}}$$

$E = 3.14\times10^{-19}\text{ J}$

$$\text{Number of photons} = \frac{\text{Power of laser}}{\text{Energy per photon}} = \frac{1.0\times10^{-3}\,\frac{\text{J}}{\text{s}}}{3.14\times10^{-19}\,\frac{\text{J}}{\text{photon}}} = 3.2\times10^{15}\,\frac{\text{photon}}{\text{s}}$$

(b) $d = 0.30\,\text{m}$

$v = c = 3.0\times10^{8}\,\frac{\text{m}}{\text{s}}$

$$t = \frac{d}{v} = \frac{0.30\,\text{m}}{3.0\times10^{8}\,\frac{\text{m}}{\text{s}}} = 1.0\times10^{-9}\text{ s}$$

(c) If each filter absorbs 96% of the photons, then 4% go through to the next filter. The final number of photons will be 4% of 4% of 4% ....

$$\begin{aligned}\text{Final number} &= (\text{original number})(0.04)^7\\ &= (3.2\times10^{15})\,(0.04)^7\\ &= 5\times10^5\text{ photons}\end{aligned}$$

(d) Time taken for one photon to travel 0.30 m = 1.0×10^{-9} s. If 5×10^5 photons emerge from the final filter every second, then we can assume that one photon emerges every $\frac{1}{5\times10^5}$ seconds. So if we take the time to travel 0.30m and divide it by the time taken between successive photons we will obtain a value that expresses the odds of two photons being in the apparatus at the same time.

$$\frac{\text{time to travel 0.30 m}}{\text{time between photons}} = \frac{1.0\times10^{-9}\text{ s}}{2.0\times10^{-6}\text{ s}} = \frac{1}{2000}$$

04. (a) $720\text{ amu} = 720(1.6605\times10^{-27}\text{ kg}) = 1.20\times10^{-24}\text{ kg}$

(b) $p = mv = (1.20\times10^{-24}\text{ kg})(210\,\frac{\text{m}}{\text{s}}) = 2.52\times10^{-22}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$

(c) $$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J.s}}{2.52\times10^{-22}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}} = 2.63\times10^{-12}\text{ m}$$

(d) $$\frac{1\times10^{-9}\text{ m}}{2.63\times10^{-12}\text{ m}} = 379$$

The molecule is about 400 times larger than its wavelength.

(e) $$\frac{50\times10^{-9}\text{ m}}{2.63\times10^{-12}\text{ m}} = 1.9\times10^{4}$$

The slits are about 19,000 times greater than the wavelength. Note that the diffraction criterion often used in high school texts is that the slit width must be the same order of magnitude as the wavelength in order to observe diffraction (and thus interference). This criterion is a crude approximation that applies to diffraction that is easy to observe. In this case, the diffraction happens, but it is very difficult to see.

(f) Using Young's equation for the double slit:

$$\Delta x = \frac{\lambda L}{d} = \frac{(2.63\times10^{-12}\text{ m})(5.0\text{ m})}{1.00\times10^{-7}\text{ m}} = 1.3\times10^{-4}\text{ m}$$

The fringes would only be a tenth of a millimetre wide!

Worksheet 04:
Advanced Math

PART 1: INVESTIGATING THE HYDROGEN ATOM

01. $E_{total\ ground} = -13.6\text{ eV} = -2.18\times10^{-18}\text{ J}$

$$E_{total} = E_{kinetic} + E_{potential} = \frac{p^2}{2m} + \left(-\frac{kq^2}{r}\right)$$

The negative sign in the parentheses indicates the attraction between the electron and the nucleus. Rearrange to solve for p

$$\frac{p^2}{2m} = E_{total} - \left(-\frac{kq^2}{r}\right)$$

$$p^2 = 2m\left(E_{total} + \frac{kq^2}{r}\right)$$

$$p^2 = 2(9.11\times10^{-31}\text{ kg})\left(-2.18\times10^{-18}\text{ J} + \frac{(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2})(1.602\times10^{-19}\text{ C})^2}{5.29\times10^{-11}\text{ m}}\right)$$

$$p = 1.994\times10^{-24}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Substitute this into the de Broglie equation

$$\lambda = \frac{\text{h}}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.994\times10^{-24}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}} = 3.324\times10^{-10}\text{ m}$$

$$\lambda = 3.32\times10^{-10}\text{ m}$$

02. $\lambda = 102.4\text{ nm}$

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\ \frac{\text{m}}{\text{s}}}{1.024\times10^{-7}\text{ m}} = 2.9297\times10^{15}\text{ Hz}$$

$$E_{photon} = hf = (6.626\times10^{-34}\text{ J}\cdot\text{s})(2.9297\times10^{15}\text{ s}^{-1}) = 1.941\times10^{-18}\text{ J}$$

$$\begin{aligned}E_{second} &= E_{ground} + E_{photon}\\ &= -2.18\times10^{-18}\text{ J} + 1.941\times10^{-18}\text{ J}\\ E_{second} &= -2.39\times10^{-19}\text{ J}\end{aligned}$$

03. Using the solutions from parts (01) and (02)

$$\begin{aligned}p^2 &= 2m\left(E_{second} + \frac{kq^2}{r}\right)\\ &= 2(9.11\times10^{-31}\text{ kg})\left(-2.388\times10^{-19}\text{ J} + \frac{(8.99\times10^9\ \frac{\text{N}\cdot\text{m}}{\text{C}^2})(1.602\times10^{-19}\text{ C})^2}{4.76\times10^{-10}\text{ m}}\right)\end{aligned}$$

$$p = 6.694\times10^{-25}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Substitute this into the de Broglie equation:

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J.s}}{6.694\times10^{-25}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}} = 9.899\times10^{-10}\text{ m}$$

$$\lambda = 9.90\times10^{-10}\text{ m}$$

04. $r_{ground} = 5.29\text{ x }10^{-11}\text{m}$

$$\text{number of wavelengths} = \frac{\text{circumference of orbit}}{\text{wavelength}}$$

$$\text{number of wavelengths} = \frac{2\pi(5.29\times10^{-11}\text{ m})}{3.324\times10^{-10}\text{ m}} = 0.9999$$

Therefore, one wavelength fits around the circumference of the ground state orbit.

$r_{second} = 4.76\text{ x }10^{-11}\text{m}$

$$\text{number of wavelengths} = \frac{2\pi(4.76\times10^{-10}\text{ m})}{9.899\times10^{-10}\text{ m}} = 3.02$$

Therefore, three wavelengths fit around the circumference of the second excited state.

PART 2: FRANCK–HERTZ EXPERIMENT

01. Energy is transferred to the electrons when work is done by the forces exerted by the field over a distance, Δx. The strength of the field is found by dividing the potential difference applied across the electrodes by the distance between them.

$$\Delta E_K = W = F \cdot d = (q\vec{\varepsilon})(\Delta x) = \frac{qV\Delta x}{L}$$

02. (a)

$$E_K = \frac{qV\Delta x}{L}$$

$$= \frac{(1.602\times10^{-19}\text{ C})(22.0\text{ V})(0.00119\text{ m})}{0.0125\text{ m}}$$

$$= 3.355\times10^{-19}\text{ J}$$

$$E_K = 3.36\times10^{-19}\text{ J}$$

(b)

$$f = \frac{E}{h} = \frac{3.355\times10^{-19}\text{ J}}{6.626\times10^{-34}\text{ J·s}}$$

$$= 5.064\times10^{14}\text{ s}^{-1}$$

$$\lambda = \frac{c}{f} = \frac{3.0\times10^{8}\text{ m/s}}{5.064\times10^{14}\text{ s}^{-1}}$$

$$= 5.924\times10^{-7}\text{ m (reddish-orange or amber)}$$

(c) The electrons are re-accelerated by the field and will gain enough energy to stimulate more photon emission further down the tube. Since the work done depends on the distance travelled under the influence of the force, the successive bands are all located at equal intervals. Note that the electron is not absorbed or destroyed during this process. The electron collides with and scatters off the gas molecule.

(d) Using the equation provided we can see that Δx is inversely proportional to V, so if we double V the spacing between the light bands will be halved.

03. $V = 13.4\text{ V}$ $\quad L = 1.25\times10^{-2}\text{ m}$ $\quad \lambda = 253\text{ nm}$

$$E = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J.s})(3.0\times10^{8}\text{ m/s})}{2.53\times10^{-7}\text{ m}}$$

$$E = 7.857\times10^{-19}\text{ J}$$

$$\Delta x = \frac{E_K L}{qV} = \frac{(7.857\times10^{-19}\text{ J})(1.25\times10^{-2}\text{ m})}{(1.602\times10^{-19}\text{ C})(13.4\text{ V})}$$

$$E = 4.575\times10^{-3}\text{ m}$$

$$\text{number of bands} = \frac{L}{\Delta x} = \frac{1.25\times10^{-2}\text{ m}}{4.575\times10^{-3}\text{ m}} = 2.7$$

So only two bands will be produced; the electrons will not have enough energy to make a third band (the answer does not round up).

04. This experiment demonstrates that the electrons have discrete energy levels. The accelerated electrons do not interact with the gas molecules until they have sufficient energy and then the interaction is a sharply defined energy transition. The electrons can only have certain discrete energy values.

05. This experiment demonstrates that the energy of light is quantized. The colour of light produced is determined by the energy lost by the valence electron as it drops from a higher energy level to a lower energy level.

PART 3: THE LIMIT OF QUANTUM OBSERVATION

01.

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{kg})(5.84\times10^{5}\text{m/s})}$$

$$= 1.2454\times10^{-9}$$

$$\lambda = 1.25\times10^{-9}\text{ m}$$

02.

$$\Delta x_{\text{interference}} = \frac{\lambda L}{d} = \frac{(1.2454\times10^{-9}\text{ m})(1.00\text{m})}{2.00\times10^{-7}\text{ m}}$$

$$\Delta x_{\text{interference}} = 6.25\times10^{-3}\text{ m}$$

03. $E_k = \frac{1}{2}k_B T_{drift}$ and $E_k = \frac{1}{2}mv_{drift}^2$

So $\frac{1}{2}k_B T_{drift} = \frac{1}{2}mv_{drift}^2$ $\qquad v_{drift} = \sqrt{\frac{k_B T_{drift}}{m}}$

04. $$\Delta x_{drift} = v_{drift} \cdot \Delta t \quad \text{and} \quad \Delta t = \frac{L}{v_{beam}}$$

$$\Delta x_{drift} = \sqrt{\frac{k_B\, T_{drift}}{m}} \cdot \frac{L}{v_{beam}}$$

$$\Delta x_{drift} = \sqrt{\frac{(1.38\times10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}})(0.250\ \mathrm{K})}{9.11\times10^{-31}\,\mathrm{kg}}} \cdot \frac{1.00}{5.84\times10^{5}\,\frac{\mathrm{m}}{\mathrm{s}}}$$

$$\Delta x_{drift} = 3.33\times10^{-3}\,\mathrm{m}$$

05. If thermal deflection, Δx_{drift}, of the electron amounts to half of the distance between interference maxima, $\Delta x_{\text{interference}}$, then the pattern will be completely washed out. See Figure 1 below.

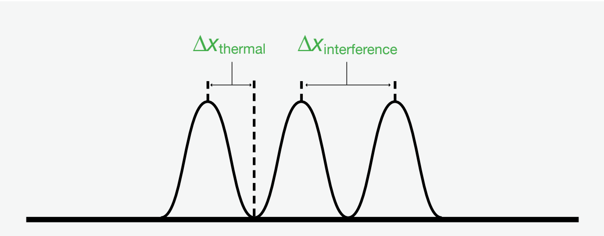


Figure 1

06. $\Delta x_{drift} = \tfrac{1}{2}\Delta x_{\text{interference}}$

$$\sqrt{\frac{\mathrm{k_B} T_{drift}}{m}} \cdot \frac{L}{v_{beam}} = \frac{1}{2}\left(\frac{\lambda L}{d}\right)$$

$$T_{drift} = \frac{1}{4}\left(\frac{\lambda^2}{d^2}\right)\left(\frac{{v_{beam}}^2 m}{\mathrm{k_B}}\right)$$

$$T_{drift} = \frac{1}{4d^2}\left(\frac{h^2}{m^2 {v_{beam}}^2}\right)\left(\frac{{v_{beam}}^2 m}{\mathrm{k_B}}\right)$$

$$T_{drift} = \left(\frac{1}{4d^2} \cdot \frac{h^2}{\mathrm{k_B}}\right)\left(\frac{1}{m}\right)$$

07. $$T_{drift} = \left(\frac{h^2}{k_{\mathrm{B}}} \cdot \frac{1}{4d^2}\right)\frac{1}{m}$$

$$T_{drift} = \left(\frac{(6.626\times10^{-34}\,\mathrm{J\ s})^2}{1.38\times10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}} \cdot \frac{1}{4(2.00\times10^{-7})^2}\right) \cdot \frac{1}{9.11\times10^{-31}\,\mathrm{kg}}$$

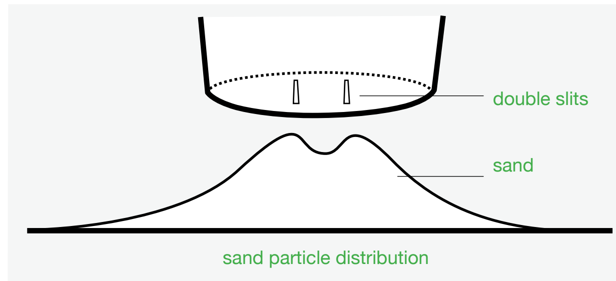
$$T_{drift} = 0.218\ \mathrm{K}$$

08. A larger object will have more mass. The increase in mass requires that the experiment be carried out with lower and lower drift temperatures and associated velocities. However, there is a practical limit to an experimenter's ability to reduce environmental interactions which cause drift velocities.

Worksheet 05:
Investigating the Nature of the Electron

PART 01: CLASSICAL PARTICLE BEHAVIOUR

01.



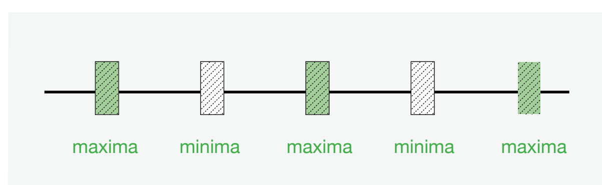
02. Answers will vary. The sand particles simply fall through and pile up beneath the slits. Both piles will be similar in height and the region between the peaks will be filled with sand from both slits. The overall shape of the profile is the additive sum of the sand particles from each slit.

03. Sand particles collide and change trajectories when they arrive at the same location at the same time.

04. Classical particles pass through one or the other slit and form a distribution, with the majority of the particles landing directly below each slit and then spilling to each side. The area between the slits is filled with particles from each slit. The overall profile is the additive sum of the particles from each slit.

PART 02: CLASSICAL WAVE BEHAVIOUR

01.



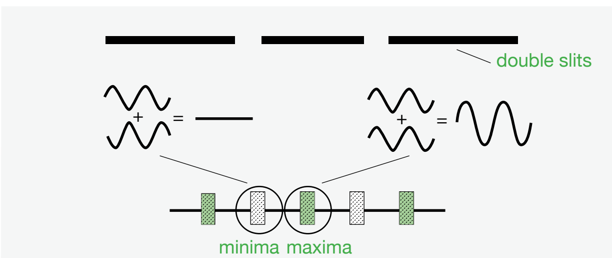
02. Answers will vary. An interference pattern is formed, with alternating regions of constructive and destructive interference radiating outward from a central region of constructive interference.

03. Waves superimpose and interfere when they arrive at the same location at the same time.

04. A single wave passes through both slits at once. The two resulting waves interfere with each other, sometimes constructively creating maxima of super crests or super troughs, and sometimes destructively cancelling each other out and creating a minima.

PART 03: LIGHT BEHAVIOUR

01.



02. Light forms an interference pattern of alternating light and dark regions. The dark regions exist in areas that would not have been dark if only one slit was open, highlighting the destructive interference of light and therefore light's wave behaviour.

PART 04: ELECTRON BEHAVIOUR

01. A prediction containing two distributions of marks concentrated behind each slit mirrors the result of sand and therefore infers an assumption that electrons will behave like classical particles. A prediction containing an interference pattern mirrors the result of light and therefore infers an assumption that electrons will exhibit wave-like behaviour.

02. The images show the formation of an interference pattern after enough electrons have passed through the apparatus. The presence of an interference pattern suggests wave-like behaviour.

03. The images are comprised of more and more individual marks, each mark representing the localized detection of an electron. The detection of individual, localized electrons illustrates particle-like behaviour.

04. Electrons are emitted as particles (although this will not be obvious to students from the data provided) and are detected as particles. This particle-like behaviour occurs whenever electrons are measured with a detector. The interference pattern that builds up over time, even if only one electron passes through the apparatus at a time, illustrates wave-like behaviour. Therefore, the electron double-slit experiment provides evidence of both particle-like behaviour and wave-like behaviour for electrons.

PART 05: SUMMARY QUESTIONS

01.

Particle Behaviour	Wave Behaviour
• localized • single particle passes through only one slit at a time • forms a two-pile distribution	• spread out • single wave passes through both slits at once • forms an interference pattern

02. Light forms an interference pattern, and in the context of a high-school double-slit experiment light is not detected as a localized object. This evidence suggests that light is best described as a wave.

03. Answers will vary. The electrons are detected as localized objects which suggest particle behaviour. However, over time an interference pattern is generated on the detection screen, suggesting wave behaviour.

04. Answers will vary. Students will likely not be comfortable with any statement that definitely describes an electron as a particle or as a wave if they have understood the apparently dual nature of the evidence.

05. Electrons exhibit both wave and particle properties.

06. Answers will vary and may hint at one of the four interpretations provided in the video.

07. Answers should present the experimental evidence for both particle and wave behaviour.

08. Answers will vary.

Who are the People in the Video

MARKUS ARNDT,
Professor, University of Vienna Arndt works in the Quantum Nanophysics Group, University of Vienna, focusing on the quantum behaviour of large molecules. Past awards include the Wittgenstein Prize, presented by Austria's Ministry for Science and Research and regarded as the country's most prestigious scientific award.

ROGER BACH,
Graduate Student, University of Nebraska-Lincoln
Bach is a student of Dr. Batelaan working towards his PhD as an experimental physicist. Amongst other projects, his is currently doing research on a variation of the electron double-slit experiment where each slit is covered in turn.

HERMAN BATELAAN,
Assistant Professor, University of Nebraska-Lincoln
Batelaan is known for his work in coherent electron control. After earning his PhD from the University of Utrecht in The Netherlands in 1991, he held positions at SUNY-Stony Brook, University of Innsbruck, UNL and the Technical University of Eindhoven.

ADAM CAPREZ,
Graduate Student, University of Nebraska-Lincoln
Caprez is an experimental physicist who completed his PhD under the supervision of Dr. Batelaan. In addition to working on the double-slit experiment with electrons, he has also done research on the relationship between electric and magnetic fields.

SARAH CROKE,
Postdoctoral Researcher, Perimeter Institute
Croke is currently exploring how to use quantum physics to process information more efficiently. She is a past recipient of a Mac Robertson travelling scholarship, which facilitated research with Dr. Cresser at Macquarie University, Australia.

STEVE FLAMMIA,
Postdoctoral Researcher, Perimeter Institute
Fascinated by all aspects of quantum computing and determined to discover the best way to build them, Flammia has worked under noted quantum information researchers Carlton M. Caves (during his PhD research) and Jian-Wei Pan (as a NSF EAPSI Scholar).

CHRIS FUCHS,
Visiting Researcher, Perimeter Institute
Fuchs has made many contributions to quantum information and quantum foundations. He was recently elected Vice-Chair of the APS Topical Group on Quantum Information and was a research staff member at Bell Labs for seven years before joining Perimeter Institute in 2007.

TIM GAY,
Professor, University of Nebraska-Lincoln
Gay is an experimental physicist currently conducting experiments with polarized electrons and probing the fundamental nature of the electron. He also performs research aimed at measuring the mass of the electron antineutrino.

GHAZAL GESHNIZJANI,
Postdoctoral Researcher, Perimeter Institute
After earning her PhD from Brown University in 2004, Geshnizjani was a postdoctoral research associate at the University of Wisconsin-Madison. She arrived at Perimeter Institute in 2007 and currently focuses on early universe cosmology.

DANIEL GOTTESMAN,
Faculty Member, Perimeter Institute Gottesman has spent over 10 years working in the field of quantum information and is widely regarded as a world expert on techniques for preventing errors in quantum computing. A former student of John Preskill, he has worked at Los Alamos, Microsoft Research, and UC Berkeley.

SEAN GRYB,
Graduate Student, Perimeter Institute and University of Waterloo Gryb is working towards his PhD in the department of physics and astronomy at the University of Waterloo, focusing on quantum gravity. In addition to research, he is committed to scientific outreach and education.

LUCIEN HARDY,
Faculty Member, Perimeter Institute Hardy has held several international research and lecturing positions. While in Rome, he collaborated on an experiment to demonstrate quantum teleportation. In 1992, he found a very simple proof of non-locality in quantum theory, now known as Hardy's theorem.

STEPHEN HAWKING,
Lucasian Professor of Mathematics, University of Cambridge and Distinguished Research Chair, Perimeter Institute Hawking is possibly the world's most famous contemporary physicist, having made several extraordinary contributions to fundamental theoretical physics. His most celebrated work was the theoretical prediction that black holes should emit radiation, known as Hawking radiation.

ROCKY KOLB,
Professor, University of Chicago Kolb is known for his work in the study of particle physics in the early universe. In addition to over 200 scientific papers, he is the author of "Blind Watchers of the Sky," an award-winning book for the general public.

RAYMOND LAFLAMME,
Director, IQC at University of Waterloo and Associate Faculty Member, Perimeter Institute
Laflamme (PhD Cambridge, 1988) began his career working with Stephen Hawking on questions in quantum gravity and cosmology, but is now a leading expert in the very different fields of quantum information theory and experiments and quantum computing.

MIKE LAZARIDIS,
PI Founder and Board Chair
In addition to initiating PI, Lazaridis is the recipient of many technology and buisness awards and, as president and co-CEO of Research In Motion, led the research and development efforts of various technological innovations including the BlackBerry, the first complete wireless email solution.

DEBBIE LEUNG,
Affiliate Member, Perimeter Institute and Faculty Member, IQC at University of Waterloo
Leung's research currently focuses on improving the efficiency of information processing using quantum physics. Leung has held postdoctoral positions at the IBM Watson Research Center, MSRI-Berkeley and IQI-Caltech, and is a CIFAR Scholar and Assistant Professor at University of Waterloo.

CHANDA PRESCOD-WEINSTEIN,
Graduate Student, Perimeter Institute and University of Waterloo Prescod-Weinstein is a doctoral student advised by Lee Smolin, a Perimeter Institute Faculty Member, and working with Niayesh Afshordi, a Perimeter Institute Postdoctoral fellow. She has a strong commitment to diversifying science and is actively involved in the National Society of Black Physicists.

ANDREW WHITE,
Professor, University of Queensland White joined the University of Queensland in 1999 from Los Alamos National Laboratory. His PhD research, conducted in Australia and Germany, won the Australian National University's Medal for best PhD thesis. White's interests are quantum information, quantum optics, and all aspects of quantum weirdness.

ANTON ZEILINGER,
Professor, University of Vienna Zeilinger has worked in top level universities and research centres around the world. In 1997, he and his colleagues confirmed aspects of quantum teleportation by teleporting light particles. He has received more than 20 awards and honorary doctorates for his work, including the Isaac Newton medal.

Appendix A:
How to Build a Black Box Model

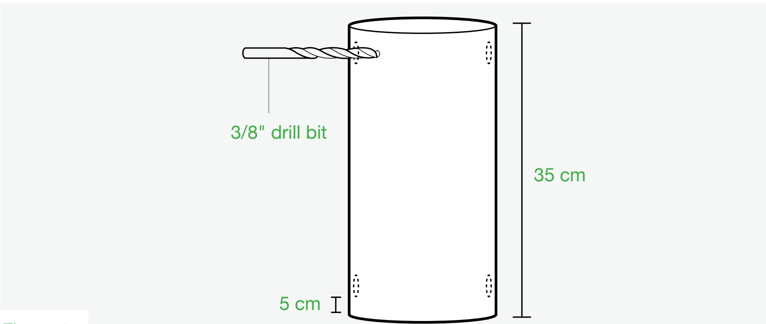


Figure 1

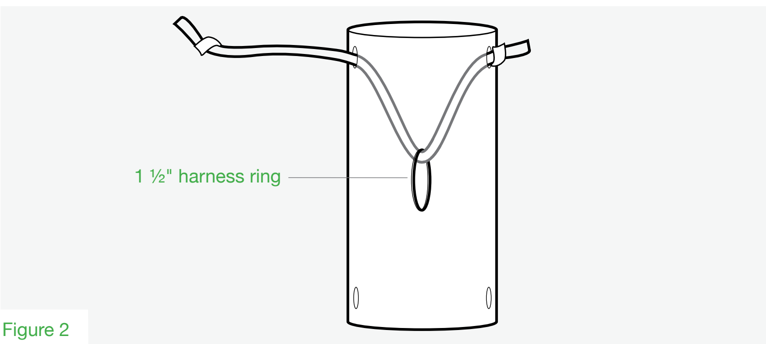


Figure 2

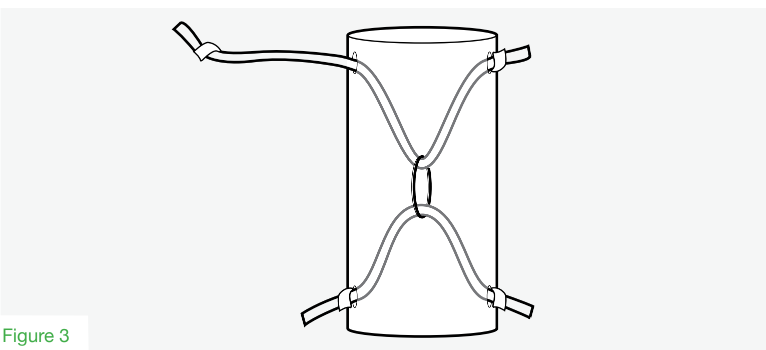


Figure 3

The Black Box Model is an effective learning tool and can be constructed with minimal cost and effort.

MATERIALS

2 pieces of 8 mm nylon rope, each 70 cm long
1 harness ring with a 40 mm diameter
1 piece of 7.5 cm diameter drainage pipe 35 cm long
2 drainage pipe end caps 7.5 cm diameter

TOOLS

3/8" drill bit
power drill

PROCEDURE

Step 1: Drill the top holes directly across from one another, each 5 cm from the top (Figure 1). Repeat for the bottom holes, each 5 cm from the bottom.

Step 2: Thread the one length of rope through the top holes and the harness ring (Figure 2).

Step 3: Tie a knot 15 cm from each end of the rope.

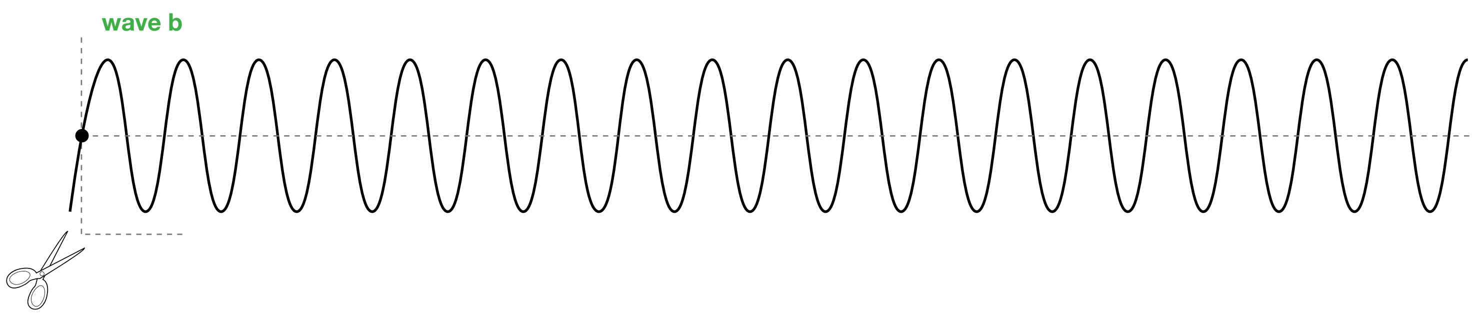
Step 4: Thread the other rope through the bottom holes. Again, ensure that the rope passes through the harness ring as indicated (Figure 3). Tie a knot 15 cm from each end of the rope.

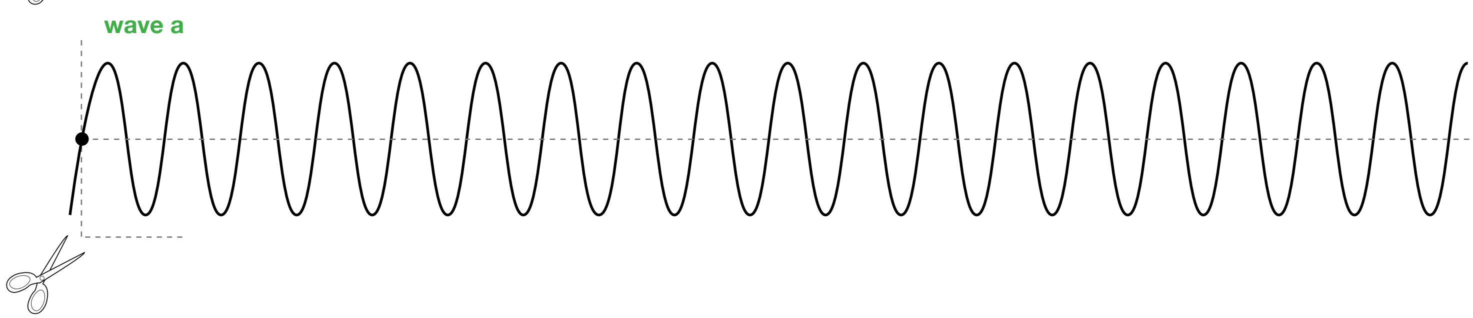
Step 5: Secure end caps.

Note: Variations on the design (without a ring for example) will enrich the discussion and work equally well. You may wish to encourage students to build their own versions of the device with bathroom tissue tubes and string, but never reveal how the device is constructed.

Appendix B:
Wave-Particle Duality Investigation

Photocopy waves a and b onto a transparency. Tape each wave to a large diameter straw and attach as shown in Figure 3 on page 15.





Appendix C:
Equations and Constants

Description	Equation		SI Unit
Path Difference	$\lvert \overline{P_nA} - \overline{P_nB} \rvert = (n - \frac{1}{2})\lambda$	$\overline{P_nA}$ = the distance from source A to a point on the nth nodal line $\overline{P_nB}$ = the distance from source B to a point on the nth nodal line λ = wavelength n = integer assigned to the nodal line	m m m
de Broglie Wavelength	$\lambda = \frac{h}{p}$	λ = wavelength h = Planck's constant p = momentum	m J·s kg·m/s
Momentum	$p = mv$	p = momentum m = mass v = velocity	kg·m/s kg m/s
Photon Energy	$E = hf$	E = energy of a photon h = Planck's constant f = frequency of the photon	J J_S S^{-1}
Young's Double Slit	$\lambda = \frac{\Delta x \cdot d}{L}$	Δx = separation of adjacent fringes λ = wavelength L = distance from slits to screen d = separation of slit centres	m m m m
Electric Potential Energy (point charge)	$E_Q = \frac{kq_1q_2}{r}$	E_Q = electric potential energy k = Coulomb's constant q_1 = electric charge on object 1 q_2 = electric charge on object 2 r = distance between object centres	J $N \cdot m^2/C^2$ C C m
Electric Potential	$V = \frac{E_Q}{q}$	V = electric potential E_Q = electric potential energy q = electric charge on object	V J C
Electric Field Intensity	$\varepsilon = \frac{F_Q}{q}$	F_Q = electric field intensity q = force exerted by field ε = electric charge on object in field	N/C N C
Electric Potential (parallel plates)	$V = \varepsilon d$	V = potential difference between parallel plates ε = electric field intensity d = distance between plates	V N/C m
Kinetic Energy	$E_K = \frac{1}{2}mv^2$	E_K = kinetic energy m = mass v = speed	J kg m/s
Franck-Hertz	$E_K = \frac{qV\Delta x}{L}$	E_K = kinetic energy of electron q = charge on electron V = potential difference applied across the tube Δx = distance between successive bands L = distance from anode to cathode	J C V m m
Thermal Drift (simplified)	$E_k = \frac{1}{2}k_BT_{drift}$	E_K = kinetic energy attributed to thermal interactions k_B = Boltzmann's constant T_{drift} = drift temperature	J J/K K

Name	Symbol	Value	SI Unit
Planck's constant	h	6.626×10^{-34}	J·s
speed of light	c	3.00×10^{8}	m/s
Coulomb's constant	k	8.99×10^{9}	$N \cdot m^2/C^2$
Boltzmann's constant	k_B	1.38×10^{-23}	J/K

Name	Symbol	Value	SI Unit
mass of electron	m_e	9.11×10^{-31}	kg
atomic mass unit	amu	1.6605×10^{-27}	kg
charge on electron	q_e	1.602×10^{-19}	C

Credits

Produced by Conrad Entertainment Inc. for Perimeter Institute for Theoretical Physics

Complete scientific, educational, production, and funding credits can be viewed on the DVD.

AUTHOR TEAM

Greg Dick
Galt Collegiate Institute and Vocational School, Cambridge, Ontario and Perimeter Institute for Theoretical Physics

Dave Fish
Sir John A Macdonald Secondary School, Waterloo, Ontario

Bruno Hartmann
Perimeter Institute for Theoretical Physics

Dr. Damian Pope
Perimeter Institute for Theoretical Physics

Roberta Tevlin
Danforth Collegiate and Technical Institute, Toronto, Ontario

SCIENCE ADVISOR

Professor Raymond Laflamme
University of Waterloo and Perimeter Institute for Theoretical Physics

EDUCATIONAL PRODUCER

Greg Dick
Galt Collegiate Institute and Vocational School, Cambridge, Ontario and Perimeter Institute for Theoretical Physics

SUPERVISING PRODUCERS

John Matlock
Director of External Relations and Outreach, Perimeter Institute for Theoretical Physics

Peter Conrad
Conrad Entertainment Inc.

DESIGNER

Jeff Watkins

EDITOR

Julia Hubble

3D IMAGES

Steve Kelly

SPECIAL THANKS TO

Professor Herman Batelaan
University of Nebraska-Lincoln

Roger Bach
University of Nebraska-Lincoln

Dr. Adam Caprez
University of Nebraska-Lincoln

Dr. Tom McCaul
Education Specialist, Denuma International

Dr. Robert Spekkens
Perimeter Institute for Theoretical Physics

IMAGE CREDITS

Tübingen Measurement Disturbance, pp. 24
Peter Sonnentag, Franz Hasselbach

Flea electron micrograph, pp. 27
Science Photo Library RF, Stockphoto Pro

PET Scan, pp. 27
Jonathan Selig/Stone/Getty Images

Black Box, pp. 37
David Dick